# Mechanics (including force, mass, and acceleration)

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## Abstract

A knowledge of classical (Newtonian) mechanics is fundamental to understanding anaesthetic equipment and the world around us. This article introduces essential concepts and illustrates them with practical examples. Topics include: Newton's Laws of Motion; instantaneous and average quantities; the relationships between distance, speed, displacement, velocity and acceleration; gravity, mass and weight; inertia and momentum; energy and power; and translational and rotational motion.

**Keywords** Acceleration; energy; force; mass; mechanics; Newton's Laws of Motion; power; work

**Royal College of Anaesthetists CPD matrix:** 1A03, 2A04, 3J00

## Learning objectives

After reading this article, you will be able to:

- describe the relationship between displacement, velocity, and acceleration
- describe Newton's Laws of Motion and give examples of their application
- describe the analogies between translational (linear) and rotational (angular) motion

Scalar quantities have magnitude only: vector quantities (indicated with a bold font or arrow, e.g. **a** or $\vec{a}$) have magnitude ($|\vec{a}|$) and direction.[1] Vectors may be added using the parallelogram law to give a *resultant* vector; resolved into mutually perpendicular *component* vectors; multiplied by scalars or using cross products to give vectors; and multiplied using dot products to give scalars.

The scalar quantity *distance* ($d$, m) describes how far an object travels from its initial position (e.g. as indicated by the odometer of a car). Motion is *uniform* if equal distances are travelled in equal intervals of time; otherwise *non-uniform* motion occurs. *Speed* ($\upsilon$, m s$^{-1}$) describes how the distance travelled by an object changes with time. *Average* speed ($v_{av}$) is given by the total distance travelled divided by the time period (i.e. $v_{av} = d/\Delta t$). Thus if you drive a distance of 100 km in 2 hours, your average speed will be 50 km h$^{-1}$. *Instantaneous* speed ($v$) at any moment is the average speed evaluated for a time interval that approaches zero (i.e. $v = \lim_{\Delta t \to 0}(\Delta d/\Delta t)$; or 'the derivative of distance') and is given by the speedometer.

In contrast, the vector quantity *displacement* ($\vec{s}$, m) describes how far an object is from its starting point in terms of the length and direction of the straight line from the initial to final position ('as the crow flies'), and is independent of actual distance travelled. *Velocity* ($\vec{v}$, m s$^{-1}$) describes how the displacement of an object changes with time (therefore 'speed is a scalar: velocity is a vector'). The speed of an object is the magnitude of its velocity: $v = |\vec{v}|$. Average velocity ($\vec{v}_{av}$) is given by the total displacement divided by the time period (i.e. $\vec{v}_{av} = \vec{s}/\Delta t$). *Acceleration* ($\vec{a}$, m s$^{-2}$) is change in velocity (i.e. a change in speed and/or direction) with time. If acceleration and velocity have the same sign (vectors in the same direction), the velocity of the object is increasing with time ('acceleration'). If acceleration and velocity have opposite signs (vectors in opposite directions), the velocity of the object is decreasing with time ('deceleration'). Instantaneous displacement, velocity and acceleration are related by change with respect to time, and their values may be found using calculus (Figure 1).

## Force and pressure

*Force* ($\vec{F}$, kg m s$^{-2}$; derived unit: newton, N) is the property which gives acceleration to a mass (i.e. causes a change in velocity of a body); 1 N is the force that gives a mass of 1 kg an acceleration of 1 m s$^{-2}$. Synonyms for force include: thrust, lift, drag, weight, load and friction. If mass is constant, a constant force (e.g. from a spring kept at constant extension) causes constant acceleration. *Pressure* (or 'stress') ($P$, kg m s$^{-2}$; derived unit: N m$^{-2}$ or pascal, Pa) is the force applied over an area. Pressure is a scalar because the forces involved are exerted equally in all directions, so there is no net direction.

## Mass, gravity and weight

The *mass* ($m$, kg) of an object is an intrinsic and unchanging property which corresponds to the quantity of matter that it contains. Mass is a measure of *inertia* (i.e. resistance to acceleration) and may be defined as the time taken for a standard force to generate a unit velocity, or $\boldsymbol{m} = \vec{F}/\vec{a}$. This is the original form of Newton's Second Law; more commonly expressed as: $\vec{F} = \boldsymbol{m}\vec{a}$. You can estimate the mass of an object by shaking it and feeling how much force is required to set it in motion — this is the principle of an *inertial balance*.

Any two spherical objects of mass $m_1$ and $m_2$ with centres of mass a distance $d$ apart will exert a mutually attractive force $\vec{F}_G$ on each other as predicted by *Newton's Law of Universal Gravitation*: $\vec{F}_G = Gm_1m_2/d^2$ (where $G$ = Newtonian Gravitational Constant $\approx 6.67 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$) (i.e. 'Matter draws matter in proportion to quantity'). The Earth (mass $\sim 5.97 \times 10^{24}$ kg; radius $\sim 6.37 \times 10^6$ m) and an apple (mass $\sim 10^{-1}$ kg; radius $\sim 5 \times 10^{-2}$ m) will therefore exert an equal and opposite force on each other of $\sim$0.98 N. However, because of the considerable difference in their mass, the resulting acceleration of the apple towards the Earth

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[1] Mass, time, distance, speed, energy (=work), pressure and power are scalars. Position (relative to a reference point), displacement, velocity, acceleration, force, momentum and impulse are vectors.

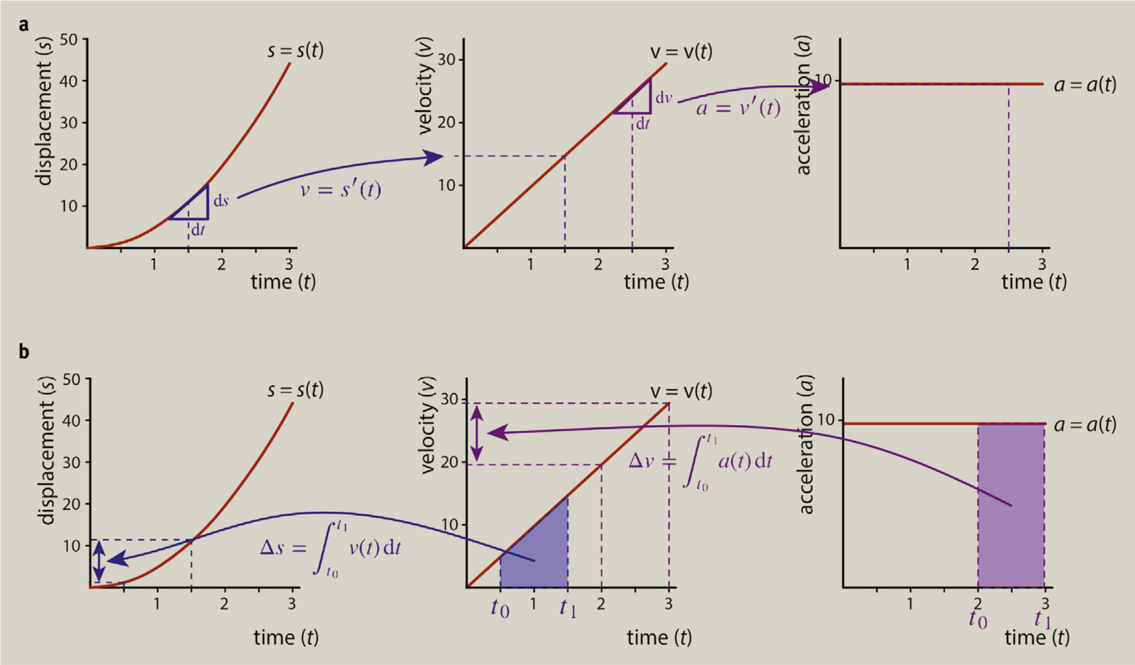

**Figure 1** Relationship between instantaneous displacement, velocity and acceleration. (**a**) Velocity ($\vec{v}$) is the rate of change (i.e. first derivative) of displacement ($\vec{s}$) with respect to time; $\vec{v} = \vec{s}'(t)$; and can be found from the gradient of the graph of displacement against time at a given time. Acceleration ($\vec{a}$) is the rate of change (i.e. first derivative) of velocity; or second derivative of displacement; with respect to time: $\vec{a} = \vec{v}'(t) = \vec{s}''(t)$. We can go further and define the jerk (or surge or lurch) as the rate of change of acceleration; and the jounce (or snap) as the rate of change of jerk. (**b**) Integration and differentiation are reciprocal processes (*the Fundamental Theorem of Calculus*). Therefore, working in the opposite direction, velocity is the area under the curve between two time points $t_0$ and $t_1$ (i.e. the definite integral) on the graph of acceleration against time; and displacement is the definite integral of velocity with respect to time.

($\sim 9.81$ m s$^{-2}$) is far greater than that of the Earth towards the apple ($\sim 1.64 \times 10^{-25}$ m s$^{-2}$). The *standard acceleration due to gravity* ($\vec{g}$, m s$^{-2}$ or N kg$^{-1}$) is the nominal gravitational acceleration of an object in a vacuum near the surface of a planet (i.e. the acceleration due to free fall neglecting air resistance); $\vec{g} \approx 9.81$ m s$^{-2}$ at the Earth's surface; and $\vec{g} \approx 1.62$ m s$^{-2}$ on the surface of the Moon. Objects of different mass (e.g. a cannonball and a feather) undergo the same acceleration due to gravity, and if dropped from the same height will therefore hit the ground simultaneously in the absence of air resistance.[2]

*Weight* ($\vec{W}$) is the force exerted on a body as a result of acceleration due gravity ($\vec{g}$). The derived SI unit of weight is therefore the newton. By Newton's Second Law, $\vec{W} = m\vec{g}$. At the same geographic location (i.e. for the same value of $\vec{g}$) mass and weight are therefore proportional to each other, with $\vec{g}$ as the constant of proportionality; and the two terms are commonly but incorrectly used interchangeably in everyday speech.[3] On the surface of the Moon, an apple will have the same mass ($\sim 1 \times 10^{-1}$ kg) as on the Earth, as the amount of matter will not change; but the weight will be $\sim 0.16$ N – i.e. 1/6 of that on Earth. If an astronaut tries to push a lunar buggy on the Moon, they will have to exert the same amount of force and expend the same amount of energy as they would if they pushed it on Earth, because the force required to produce a given acceleration is dependant on mass, not weight. Weighing scales that use the extension of a spring or change in electrical properties of a load cell measure force (weight), and will therefore read a lower value on the Moon. A pan balance measures the weight of an object relative to a reference weight at the same geographical location. It therefore measures mass, and will give the same reading on the Moon as on Earth.

It is impossible for an isolated observer (and most scientific instruments) to distinguish whether acceleration is due to movement or gravity. Both are perceived as forces with apparent direction opposite to that of actual acceleration. The '*g-force*' experienced by fighter pilots is the perceived force per unit mass (i.e. perceived weight) as a result of acceleration due to movement, expressed as a multiple of ($\vec{g}$). Astronauts in close Earth orbit 100 km above the Earth's surface experience apparent weightlessness ('zero g'), however $\vec{g}$ is essentially normal at this altitude ($\approx 9.5$ N kg$^{-1}$): the effect occurs because the orbit trajectory causes both astronauts and spacecraft to continually fall towards Earth with the same constant acceleration.

*Newton's Laws of Motion* (N1–3) describe the relationship between force and motion, and are defined in the lower section of Table 1. In brief, they describe the following concepts:

- N1: Inertia: Forces change motion
- N2: $\vec{F} = m\vec{a}$
- N3: Action–reaction: Forces come in equal and opposite pairs

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[2] Theorized by Galileo in 1589, and subsequently demonstrated by John Miller in 1761 using an evacuated glass tube, and by Cdr David Scott on the Moon on 2 Aug 1971 during the Apollo 15 mission.

[3] You weigh 0.5% less at the equator ($\vec{g} \approx 9.78$ N kg$^{-1}$) than at the poles ($\vec{g} \approx 9.83$ N kg$^{-1}$) due to centripetal forces arising from the Earth's spin, and the fact that the Earth is slightly fatter at the equator.