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Short communication

Can back-projection fully resolve polarity indeterminacy of independent component analysis in study of event-related potential?

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ABSTRACT

In the study of event-related potentials (ERPs) using independent component analysis (ICA), it is a traditional way to project the extracted ERP component back to electrodes for correcting its scaling (magnitude and polarity) indeterminacy. However, ICA tends to be locally optimized in practice, and then, the back-projection of a component estimated by the ICA can possibly not fully correct its polarity at every electrode. We demonstrate this phenomenon from the view of the theoretical analysis and numerical simulations and suggest checking and modifying the abnormal polarity of the projected component in the electrode field before further analysis. Moreover, when several components are to be projected, instead of the parallel projection of those components simultaneously, the sequential projection of component by component permits the correction of the abnormal polarity of a certain projected component at a certain electrode, which can improve the accuracy of the back-projection. Furthermore, after one extracted component by the ICA is projected back to electrodes under the global optimization, we cannot achieve the real source yet, but the determined scaled source, i.e., the multiplication between the real source and the mapping coefficient from the source to the point at the scalp.

1. Introduction

Independent component analysis (ICA) [1] has been extensively used in the study of event-related potentials (ERPs) [2]. It is known that the independent components (ICs) estimated by ICA tend to possess the magnitude and polarity (positive or negative) indeterminacy [1]. Because the peak amplitude is one of the most important parameters to describe an ERP [3], the correction of such ambiguity of an IC is necessary to the study of ERPs. Thus, a back-projection of the desired ERP component to the electrode field often follows the ICA decomposition [4]. For example, as Makeig and colleagues noted, the projection of the *i*th IC onto the original data channels is given by the outer product of the *i*th row of the component activation matrix with the *i*th column of the inverse unmixing matrix, and is in the original units (e.g., microvolts) [4]. However, despite of the vital influence of the back-projection to the further study of ERPs, the performance of the back-projection has never been deeply analyzed except by our previous report [5].

This study attempts to analyze the mathematical composition of the back-projection of ICs under the global and local optimization of the ICA decomposition [1,6,7], respectively. Particularly, the polarity is important to identify an ERP, for example, N1 is the negative peak and P3 is the positive peak [3] under certain reference. This study mainly discusses the polarity of the back-projection of an estimated ERP-like IC in the electrode field.

2. Abnormal polarity of projected components at electrodes

2.1. General solution of ICA on EEG

The classic ICA decomposition on EEG may be illustrated as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t), \tag{1}$$

$$\mathbf{y}(t) = \mathbf{W}\mathbf{x}(t), \tag{2}$$

where $\mathbf{x}(t) = [x_1(t), \ldots, x_N(t)]^T$ denotes available multichannel EEG recordings at the scalp, $\mathbf{s}(t) = [s_1(t), \ldots, s_M(t)]^T$ represents unknown sources of the brain, $\mathbf{A}$ with full rank contains the unknown mapping coefficients from any source to any electrode, $\mathbf{W}$ exhibits the unmixing matrix, and $\mathbf{y}(t) = [y_1(t), \ldots, y_M(t)]^T$ manifests the extracted components by the ICA. The ICA decomposition is to find the unmixing matrix through minimizing or maximizing some cost functions [1,7]. Without loss of generality, we assume N sources and N sensors in this study.

Usually, after the unmixing matrix is obtained, its inverse is often computed to recover of original units of recordings. The projection matrix is simply obtained by

$$\mathbf{B} = \mathbf{W}^{-1}. \tag{3}$$

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This is because every column of the projection matrix includes the relative projection strengths of the corresponding component onto all of the scalp sensors [4]. Furthermore, the accuracy of such projection depends on the performance of the ICA decomposition.

2.2. Global matrix and projection matrix

One method to evaluate the performance of ICA is to investigate its global matrix **C** [7]. Then, the ICA solution in Section 2.1 can be interpreted as below

$$\mathbf{C} = \mathbf{WA}. \tag{4}$$

$$\mathbf{y}(t) = \mathbf{Cs}(t). \tag{5}$$

$$\mathbf{B} = \mathbf{AC}^{-1}. \tag{6}$$

When only one nonzero element exists in each row and each column of the global matrix **C**, the performance of the ICA is globally optimized; otherwise it is locally optimized.

2.2.1. Global optimization and projection matrix

Under the global optimization, **C** can be decomposed into a permutation matrix **P** and a diagonal matrix **D**, i.e.,

$$\mathbf{C} = \mathbf{PD}. \tag{7}$$

Thus, the estimated components turn to be

$$\mathbf{y}(t) = \mathbf{PDs}(t). \tag{8}$$

Explicitly, Eq. (8) means that every estimated component is scaled version of the corresponding source and does not contain the information of other sources.

Furthermore, the projection matrix becomes

$$\mathbf{B} = \mathbf{AD}^{-1}\mathbf{P}^{-1}. \tag{9}$$

Because $\mathbf{D}^{-1}$ and $\mathbf{P}^{-1}$ are also the diagonal and permutation matrixes respectively, every column of the projection matrix **B** corresponds to the permuted and scaled version of the relevant column of the mapping matrix **A**.

2.2.2. Local optimization and projection matrix

Under the local optimization, the global matrix cannot be factorized into the multiplication of the permutation matrix **P** and the diagonal matrix **D**, hence, Eqs. (7)–(9) become inequations, i.e., $\mathbf{C} \neq \mathbf{PD}$, $\mathbf{y}(t) \neq \mathbf{PDs}(t)$, $\mathbf{B} \neq \mathbf{AD}^{-1}\mathbf{P}^{-1}$.

Consequently, every estimated component is still the mixture of some sources, and every column of the projection matrix **B** is the mixture of randomly scaled relative projection strengths of some components onto all of the scalp sensors. In this case, the estimated components and the projection matrix can only be obtained from Eqs. (2) and (3). Since such a case may appear with very high probability using ICA to extract ERPs [5,6], we are interested in studying the projection under the local optimization.

2.3. Project one component back to electrodes

The projection of the kth IC at the ith electrode can be described as

$$e_{ik}(t) = b_{ik}y_k(t), \tag{10}$$

where b_{ik} is the ith element of the kth column of **B** and $y_k(t)$ is the kth component of $\mathbf{y}(t)$.

2.3.1. Projection under global optimization

Under the global optimization, based on Eq. (8), the estimated component can be described as

$$y_k(t) = p_{km}d_{mm}s_m(t) = d_{mm}s_m(t), \tag{11}$$

where p_{km} with value '1' in a permutation matrix is the nonzero element of **P** at the kth row and the mth column, d_{mm} is the mth diagonal element of **D**, and $s_m(t)$ is the mth element of $\mathbf{s}(t)$. Based on Eq. (9), the projection coefficients at the kth column of **B** and the ith electrode can be interpreted as

$$b_{ik} = a_{im}\frac{1}{d_{mm}}p_{mk} = \frac{a_{im}}{d_{mm}}. \tag{12}$$

Substituting Eqs. (11) and (12) into Eq. (10), we obtain the projection of the kth component at the ith electrode as below,

$$q_{ik}(t) = a_{im}s_m(t), \tag{13}$$

where, to discriminate the projection under the local optimization, $q_{ik}(t)$ is used for the notation instead of $e_{ik}(t)$ for the projection under the global optimization.

2.3.2. Projection under local optimization

Under the local optimization, the estimated component derived from Eq. (5) can be illustrated as

$$y_k(t) = \sum_{j=1}^{N} c_{kj}s_j(t), \tag{14}$$

where some c_{kj} (j is the element of $\{1, \ldots, N\}$) may be zero and this depends on the performance of the ICA. Substituting Eq. (14) into (10), we achieve the projection of the kth component at the ith electrode under the local optimization as the following

$$e_{ik}(t) = b_{ik}\sum_{j=1}^{N} c_{kj}s_j(t) = b_{ik}c_{km}s_m(t) + b_{ik}\sum_{j=1, j\neq m}^{N} c_{kj}s_j(t). \tag{15}$$

The right part of Eq. (15) is separated into two items because $b_{ik}c_{km}s_m(t)$ can dominate $e_{ik}(t)$ under the satisfactory ICA performance, i.e., in the kth row of the global matrix **C**,

$$\left|c_{km}\right| >> \left|c_{kj}\right|, \tag{16}$$

where $|\bullet|$ denotes the absolute value of a scalar.

Under the assumption of Eq. (16), to a certain source, $b_{ik}c_{km}$ can determine the polarity and can almost determine magnitude of the projected component in the electrode field. Under the global optimization, $b_{ik}c_{km}$ in Eq. (15) is equal to the mapping coefficient a_{im} in Eq. (13); however, under the local optimization, both c_{km} and b_{ik} do not obey any essential relationship as illustrated in Section 2.2.2, moreover, they may have random polarities, respectively, hence, the sign of $b_{ik}c_{km}$ may be different with that of a_{im}, then, the polarity of the projected component $e_{ik}(t)$ may be opposite to that of the mapping component $q_{ik}(t)$. In such a case, we define that the abnormal polarity happens [5].

To correct the abnormal polarity, the projected component $e_{ik}(t)$ can be multiplied by '−1'. Indeed, such correction does not change the sign of the 'actual' brain sources. Under the local optimization, Eq. (15) is the approximation of Eq. (13) under the assumption mentioned above. This means there are errors between $q_{ik}(t)$ and $e_{ik}(t)$. Such errors are just originated from the separation matrix **W** in Eq. (2). Under the global optimization, as illustrated by Eqs. (8) and (11), the separation matrix **W** can guarantee that only one estimated component corresponds to one source. However, under the local optimization, as illustrated by Eq. (14), the separation matrix **W** cannot guarantee that only one estimated component contains the information of one source. Furthermore, since the projection matrix **B** is the inverse of the separation matrix, it contains errors too. The correction made in this study is indeed the post-processing after the ICA, and it can also be regarded as the further processing on certain element of the projection matrix **B**. So, the correction does not change the real source in the brain.