Biomedical Signal Processing and Control 4 (2009) 162–169

Contents lists available at ScienceDirect

Biomedical Signal Processing and Control

journal homepage: www.elsevier.com/locate/bspc

Short communication

Methods of weighted averaging of ECG signals using Bayesian inference and criterion function minimization

Alina Momot

Silesian University of Technology, Institute of Computer Science, 16 Akademicka St., 44-100 Gliwice, Poland

A R T I C L E I N F O

Article history:
Received 26 October 2008
Received in revised form 23 December 2008
Accepted 14 January 2009
Available online 14 February 2009

Keywords:
ECG signal
Weighted averaging
Bayesian inference
Criterion function minimization

A B S T R A C T

Averaging signals in time domain is one of the main methods of noise attenuation in biomedical signal processing in case of systems producing repetitive patterns such as electrocardiographic (ECG) acquisition systems. This paper presents a comprehensive study of weighted averaging of ECG signal. Presented methods use criterion function minimization, partitioning of input set of data in the time domain as well as Bayesian and empirical Bayesian framework. The existing methods are described together with their extensions. Performance of all presented methods is experimentally evaluated and compared with the traditional averaging by using arithmetic mean and well-known weighted averaging methods based on criterion function minimization (WACFM).

1. Introduction

It can be noticed that certain biological systems produce repetitive patterns. In these cases averaging in the time domain may be used for the purpose of noise attenuation. Traditional arithmetic averaging technique assumes the constancy of the noise power cycle-wise, however the most types of noise are not stationary. In reality the variability of noise power from cycle to cycle is observed, which constitutes a motivation for using methods of weighted averaging. The idea behind these methods is to reduce influence of hardly distorted cycles on resulting averaged signal (or even eliminates them).

One of biomedical signals with a quasi-cyclical character is the electrocardiographic (ECG) signal, which is a recording of the electrical activity of the heart over time. In this signal different kinds of disturbances are present, such as 50 Hz (or 60 Hz) powerline interference, baseline wandering, electromyographic noise or motion artifacts (from the varying electrode-skin contact impedance caused by the electrode movement). Noise attenuation can be achieved by various methods such as wavelet discrimination [1], principal component and independent component techniques [2], robust principal component analysis [3]. Regarding repetitive patterns of the ECG signal, averaging in the time domain may also be used for the purpose of noise attenuation, and even in some medical applications the averaging is the only method taken into account. The computing of averaged ECG beats is necessary for the aim of evaluating some clinical indexes based on the ST depression such as ST versus HR (ST/HR) diagram as well as ST/HR hysteresis, which integrates ST/HR diagram during both exercise and recovery phases of the stress test [4].

Another example of the need of use signal averaging is the high resolution electrocardiogram, which detects very low amplitude signals in the ventricles called late potentials in patients with abnormal heart conditions. Due to their very low magnitudes, late potentials are not visible in a standard ECG. Detecting ventricular ECG signals of very low magnitudes called ventricular late potentials (VLP) requires averaging a number of signals. The high resolution electrocardiogram currently requires ensemble averaging of 200–600 beats to produce a high fidelity signal estimate [5], which corresponds to the duration of the test from 3 to 8 min. However, instead of averaging N successive cardiac complexes (recorded with one electrode), there were conducted studies on averaging the same cardiac complex recorded at N different locations on body surface (by N different electrodes) [6].

Noise attenuation by signal averaging is also used in the case of estimating brain activity evoked by a known stimulus. The number of averages needed to obtain a reliable response ranges from a few dozen for visual evoked potentials (VEP) to a few thousand for brainstem auditory evoked potentials (BAEP), which corresponds to long duration of the test and therefore the assumption of constancy of noise is not hold. In order to shorten a typical EP session several algorithms have been developed which improve the signal to noise ratio of the averaged evoked potential, such as applying filters to the average response [7] or applying the weighted averaging [8]. There are numerous approaches to weighted averaging, such as methods based on the minimum energy principle [8,9], Kalman filter theory [10], adaptive

E-mail address: alina.momot@polsl.pl.

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doi:10.1016/j.bspc.2009.01.002

estimation of the weights [11], criterion function minimization [12] or Bayesian inference [13]. Finding the solution of a optimization problem is the crucial part of many of these methods. It can be formulated as the minimization of a certain distance between the reference signal and the averaged signal (for some distance function) or the maximization of a posterior probability over the signal (and possibly some other parameters), in case of a probabilistic approach.

The paper presents a comprehensive study of methods for resolving of the signal averaging problem, there are presented two types of methods together with their extensions. The first of them exploits the criterion function minimization technique and partitioning of input set of a data in the time domain. The second method incorporates Bayesian and empirical Bayesian inference and the expectation-maximization technique.

There is also presented performance comparison of all described methods using a synthetic ECG signal from CTS database as well as a real ECG signal in presence of synthetic (Gaussian and Cauchy) or real muscle noise. It is also worth noting that the presented methods can be use not only for weighted averaging ECG signal but for any signal which is quasi-repetitive and synchronized.

2. Weighted averaging

After segmentation, synchronization and elimination of ectopic beats, each cycle is represented by

$$y_i(j) = x(j) + n_i(j), \tag{1}$$

where i is the cycle index $i \in \{1, 2, \ldots, M\}$, and j is the time index in the single cycle $j \in \{1, 2, \ldots, N\}$ (all cycles have the same length N). Thus each signal cycle $y_i(j)$ is the sum of the deterministic (useful) signal $x(j)$, which is the same in all cycles, and the random noise $n_i(j)$ with zero mean and variance for the ith cycle σ_i^2. The noise process remains stationary within each evolution, but its variance may vary from one beat to the other.

The weighted average is given by

$$v(j) = \sum_{i=1}^{M} w_i y_i(j), \tag{2}$$

where w_i is the weight for ith signal cycle and $v(j)$ is the averaged signal. Generally, the assumption

$$\sum_{i=1}^{M} w_i = 1 \tag{3}$$

is taken, which leads to the unbiased estimation.

In the case of samples with possibly unequal variances the classical procedure for weighted average leads to the weights which are proportional to the inverses of corresponding variances [14]:

$$w_i = \frac{\sigma_i^{-2}}{\sum_{m=1}^{M} \sigma_m^{-2}}. \tag{4}$$

The variance of the averaged data is given by

$$\mathrm{Var}(v(j)) = \left(\sum_{i=1}^{M} \frac{1}{\sigma_i^2} \right)^{-1}, \tag{5}$$

while the variance of the arithmetically averaged data is given by

$$\mathrm{Var}(v(j)) = \sum_{i=1}^{M} \frac{\sigma_i^2}{M^2}. \tag{6}$$

If the noise power is the same in all cycles (σ^2), both arithmetically and weighted averaged signal have the same variance:

$$\mathrm{Var}(v(j)) = \frac{\sigma^2}{M}. \tag{7}$$

Assuming that the noise variances σ_i^2 in each cycle $i \in \{1, 2, \ldots, M\}$ are known, the weights can be explicitly computed from (4). However, it is impossible to measure the variances directly. Thus the noise variances are estimated or there are employed methods to obtain the optimal weights without direct estimation of the noise variance. Both of these approaches will be presented below. Estimation of the noise variations is performed in the Bayesian method. By contrast, in the weighted averaging using criterion function minimization direct estimation of the noise variances is not necessary.

3. Methods using criterion function minimization

Below there will be presented two-weighted averaging methods, which are based on the minimization of a certain distance between the reference signal and the averaged signal (for some distance function). The weighted averaging method WACFM [12], which will be treated as the reference method in numerical experiments and proposed new weighted averaging method using criterion function minimization and based on partitioning of input set in time domain. It is a generalization of method proposed in [15], where the input set was divided into two separate subsets while the new method allows to divide the input set into any arbitrarily chosen number of disjoint subsets.

3.1. WACFM method

In 2002, Leski proposed weighted averaging method based on criterion function minimization (WACFM) [12]. The idea of the method is based on the fact that for $\boldsymbol{y}_i = [y_i(1), y_i(2), \ldots, y_i(N)]^{\mathrm{T}}$, $\boldsymbol{w} = [w_1, w_2, \ldots, w_M]^{\mathrm{T}}$ and $\boldsymbol{v} = [v(1), v(2), \ldots, v(N)]$ minimization the following scalar criterion function:

$$I_m(\boldsymbol{w}, \boldsymbol{v}) = \sum_{i=1}^{M} (w_i)^m \rho(\boldsymbol{y}_i - \boldsymbol{v}), \tag{8}$$

where $\rho(\cdot)$ is a measure of dissimilarity for vector argument and $m \in (1, \infty)$ is a weighting exponent parameter, with respect to the weights vector yields:

$$\forall\, i \in \{1, 2, \ldots, M\} \quad w_i = \frac{\rho(\boldsymbol{y}_i - \boldsymbol{v})^{1/(1-m)}}{\sum_{k=1}^{M} \rho(\boldsymbol{y}_k - \boldsymbol{v})^{1/(1-m)}}. \tag{9}$$

When the quadratic function $\rho(\boldsymbol{t}) = \|\boldsymbol{t}\|^2 = \boldsymbol{t}^{\mathrm{T}}\boldsymbol{t}$ is used, the averaged signal can be obtained as

$$\boldsymbol{v} = \frac{\sum_{i=1}^{M} (w_i)^m \boldsymbol{y}_i}{\sum_{i=1}^{M} (w_i)^m}, \tag{10}$$

for the weights vector given by (9) with the quadratic function. The optimal solution for minimization (8) with respect to $\boldsymbol{w}$ and $\boldsymbol{v}$ is a fixed point of (9) and (10) and could be obtained from the Picard iteration.

Therefore the weighted averaging algorithm can be described as follows, where ε is a preset parameter:

1. Fix $m \in (0, \infty)$. Initialize $\boldsymbol{v}(0)$. Set the iteration index $k = 1$.
2. Calculate $\boldsymbol{w}^{(k)}$ for the k th iteration using

$$\forall\, i \in \{1, 2, \ldots, M\} \quad w_i^{(k)} = \frac{\|\boldsymbol{y}_i - \boldsymbol{v}^{(k-1)}\|^{2/(1-m)}}{\sum_{j=1}^{M} \|\boldsymbol{y}_j - \boldsymbol{v}^{(k-1)}\|^{2/(1-m)}}. \tag{11}$$

3. Update the averaged signal for the k th iteration $\boldsymbol{v}^{(k)}$ using (10) and $\boldsymbol{w}^{(k)}$.