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High resolution time-frequency representation for chirp signals using an adaptive system based on duffing oscillators

Antonio H. Costa [a,*], Rogerio Enríquez-Caldera [b], Maribel Tello-Bello [b], Carlos R. Bermúdez-Gómez [b]

[a] Department of Electrical and Computer Engineering, University of Massachusetts, Dartmouth, MA 02747, USA
[b] National Institute of Astrophysics, Optics and Electronics INAOE, Puebla, Mexico

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ABSTRACT

This paper presents a novel methodology to estimate the frequency shift in chirp signals with SNRs as low as −17 dB through the use of an adaptive array of Duffing oscillators. The system used here is an array of five Duffing oscillators with each oscillator's response enhanced through a correlation with the reference signal. As a final result, a time-frequency depiction is provided by the Duffing array for further analysis of chirp signals.
Using computer simulated experiments, it is found that the analysis of chirp signals with low SNR by means of the Duffing oscillator shows a markedly better performance than the conventional methods of time-frequency analysis. To this end, the results obtained from the proposed Duffing method are compared against some recent techniques in time-frequency analysis.
Furthermore, to strengthen the proposed representation, Monte Carlo simulation is used.

1. Introduction

Typically, detection and estimation of time-varying signals is done through time-frequency (TF) methodologies like those of short-time Fourier Transform, discrete wavelet transform and other more modern techniques like the Wigner and the Choi–Williams distributions [1]. However, all these TF techniques exhibit difficulties when the time-varying signals under study have low SNRs, as in the case of chirp signals immersed in noise.

In recent years, detection of extraordinarily low SNR signals with a constant frequency has been reported using chaotic oscillators [2–5] and specifically the Duffing oscillator [6–11]. The use of an array of Duffing oscillators has also permitted the detection of nonlinear time-varying frequencies under high levels of noise – with better results than those obtained from conventional TF techniques – working with chirp signals in environments with very low SNRs [12]. This array approach presented a drawback due to its imprecise way of measuring changes in frequency.

In contrast, traditional and modern TF analysis techniques when used to measure signals whose frequency changes over time including the spectrogram, the continuous wavelet transform [13–15] and the Wigner distribution [16–18] have given very accurate results in high SNR scenarios, but they are all seriously impacted when the SNR is too low.

Here, and due to an improvement in measuring and detecting the transitions between the periodic and chaotic states of an array of five, self-adjusting Duffing oscillators, this paper proposes a novel method for the analysis of chirp signals with very low SNR in the TF domain as an advantaged choice to the Choi–Williams distribution [1], and the Multiform Tiltable Exponential Distribution (MTED) [18].

Furthermore, the experimental comparison allowed noticing that there exist two inner limitations related to the Duffing oscillator: i) Despite the published claims that the chaotic oscillator is immune to noise [6,9,11,14,19–23], it has a noise threshold under which the oscillator can work as a good detector; and ii) the array system oscillator also has a measuring threshold for the frequency variation ratio present in the chirp signal.

In what follows, Section 2 provides a short description of the Duffing oscillator chaotic behavior whereas Section 3 describes how such behavior is used to detect chirp signals in high levels of noise and how to obtain the corresponding parameter measurements in the most precise form. Section 4 describes the proposed adaptive system that allows for the measurement of the instantaneous frequency variation of a highly dynamic single component chirp signal within a large frequency range. Finally, Section 5 shows the experimental comparison, based on the relative MSE,

* Corresponding author.
E-mail address: Acosta@umassd.edu (A.H. Costa).

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among the TF representations using the Choi–Williams distribution, the MTED and the proposed Duffing Adaptive System.

2. Duffing oscillator operation

The general Duffing oscillator can be modeled as the following non-linear differential equation [24]:

$$\ddot{y} + 2\zeta\dot{y} + \mu\dot{y}^3 + \alpha y + \gamma y^3 = 0 \tag{1}$$

where y represents displacement, ζ is the damping ratio, μ is the non-linear damping coefficient, α is the linear stiffness and γ is the non-linear stiffness.

Analysis of the Duffing system under no external force using the homogeneous equation (1) gives 3 equilibrium points, one equilibrium point at $(y_{eq}, \dot{y}_{eq}) = (0, 0)$, and two equilibrium points $(y_{eq}, \dot{y}_{eq}) = (\pm\sqrt{-\frac{\alpha}{\gamma}}, 0)$ under two different conditions [24]:

i) One condition occurs when the stiffness coefficients, both linear and non-linear, have the same sign, that is $\alpha\gamma > 0$.
ii) The other condition occurs when both stiffness coefficients have different signs, that is, $\alpha\gamma < 0$.

Such analysis determines that the Duffing oscillator has a chaotic behavior if and only if all three equilibrium points are present. Further analysis shows that under all the above conditions, the Jacobian evaluated at the corresponding stability points resulted in the non-linear damping element becoming null, that is, $\mu\dot{y}_{eq}^3 = 0$.

Thus, working under such conditions, and when we apply an exciting force composed by the sum of two sinusoidal parts in the presence of additive noise $n(t)$, equation (1) becomes

$$\ddot{y} + 2\zeta\dot{y} - \alpha y + \gamma y^3 = F_r\cos(\omega t) + A\cos\big[(\omega + \Delta\omega)t + \varphi\big] + n(t) \tag{2}$$

where F_r is the amplitude of a given single reference signal with the frequency ω chosen to be equal to the initial frequency of an applied input signal (amplitude A, arbitrary phase φ and a frequency drift from the reference of $\Delta\omega$). The additive noise has standard deviation σ.

The operating principle of the oscillator is based on the frequency difference between the two involved signals in the equation: the proper reference signal of the Duffing Oscillator ($F_r\cos(\omega t)$) and the introduced external signal ($A\cos[(\omega + \Delta\omega)t + \varphi]$).

A complete mathematical demonstration of such transitions is developed in [8] where it is also shown that the amplitude of the oscillator's response is given by

$$F(t) = \sqrt{F_r^2 + 2F_r\cos(\Delta\omega t + \varphi) + A^2}. \tag{3}$$

When $F(t)$ is smaller than a given but fixed Fo the oscillator exhibits a chaotic state, and when $F(t)$ is bigger than Fo the oscillator presents its periodic state. Thus, Fo establishes a threshold for transitions between chaos and periodicity [23]. Furthermore, the frequency difference $\Delta\omega$ can be estimated when calculating the time at which those transitions occur by means of

$$\Delta T = \frac{2\pi}{\Delta\omega}. \tag{4}$$

It has also been shown in [20] that any possible noise added to the system, does not affect such transitions and only affects the trajectory of the response, re-enforcing in this manner the chaotic intermittence behavior.

From equation (4), the period at which the transitions occur is inversely proportional to $\Delta\omega$, which allows a precise frequency measurement of the input signal even in the presence of noise. Therefore, measuring the period ΔT is one of the most important steps along the process.

It should be noted that the existing force for this case only contains a single reference signal, thus limiting this development to single component signal applications.

With the purpose of detecting a signal with any frequency variation without necessarily modifying any parameter in equation (1) when ω varies, it is convenient to apply a variable transformation [10] to the system equation, obtaining the state-space system described by

$$\begin{cases} \dot{x} = \omega y \\ \dot{y} = \omega\big(-2\zeta y + \alpha x - \gamma x^3 + F_r\cos(\omega t)\big) \end{cases} \tag{5}$$

It is worth noticing that the state equations (5) have the angular frequency ω as a factor and, therefore, the amplitude in the oscillator response increases as ω increases [4,10,21,23] and this may cause a variation in the threshold between the two possible states. This, in turn, may cause the oscillator to fall off the chaotic intermittence, making it impossible to estimate any dynamic frequency changes in the incoming signal. This has to be taken into consideration when attempting to detect chirp signals.

In other words, frequency measurements work well for stationary frequency signals. However, our motivation is to verify that the Duffing oscillator can permit the measurement of time dependent frequency signals, specifically chirp signals.

3. Chirp detection with the Duffing oscillator

This section is intended to give the reader an idea of how it is possible to detect chirp signals using the Duffing oscillator, the main contribution of this research. We also explain how the system generates a time-frequency representation (TFR). It is important to note here that the proposed system has the ability to detect linear and nonlinear frequency variations and that these variations represent accelerations and decelerations or even changes in acceleration which, in fact, are more consistent with the Doppler Effect in real situations.

The Duffing oscillator working under a reference chirp signal whose frequency variation in time, represented by $\dot{\omega}$, is described by

$$\ddot{y} + 2\zeta\dot{y} - \alpha y + \gamma y^3 = F_r\cos\big(\omega t + \dot{\omega}t^2\big) + A\cos\big[(\omega + \Delta\omega)t + (\dot{\omega} + \Delta\dot{\omega})t^2 + \varphi\big] + n(t). \tag{6}$$

To work with this type of signal it was assumed that the Duffing oscillator is able to accurately detect the frequency of signals with linear phase variations operating under chaotic conditions as discussed under equation (5). Thus, it is possible to conceive a new time approach where the chirp signal is divided into small time windows short enough to assume that, within each window, the frequency can be considered constant. Furthermore, the selection of the time window has to ensure that the variation of frequency does not take the oscillator out of the intermittence condition.

To calculate the accepted frequency variation present in a windowed linear chirp signal described by

$$X(t) = \begin{cases} \cos(\omega t + \dot{\omega}t^2) & \text{for } t < |\frac{T}{2}| \\ 0 & \text{otherwise} \end{cases} \tag{7}$$