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Performance improvement of direction finding algorithms in non-homogeneous environment through data fusion

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ABSTRACT

This paper proposes a new effective approach to improve the performance of DOA (Direction Of Arrival) algorithms when bursts affect the array data. The proposed approach is based on the combining of data fusion techniques and the results of theoretical performance analysis of conventional DOA algorithms. For this purpose, the received array data is first split in M time-segments. Then, the DOAs are estimated from each data segment using a conventional DOA algorithm. The obtained estimates are fused using the federated fusion algorithm according to their statistical accuracy obtained from the well-documented performance analysis of the considered algorithm. As proof of concept of the proposed approach, numerical experiments have been conducted by considering the MUSIC algorithm. The obtained results show that the new algorithm outperforms the conventional one in terms of accuracy in a non-homogeneous environment. Therefore, it exhibits enhanced robustness capability. Moreover, it reduces the memory cost and computational complexity which makes it suitable for real time applications. To our knowledge, it is the first time that theoretical performance analysis results are exploited for the derivation of new subspace-DOA methods.

1. Introduction

Direction finding and spectral estimation are challenging issues that involve subspace processing techniques. Typical methods are the Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) [1,2] and Multiple SIgnal Classification (MUSIC) algorithm [3,4]. So far, many variants of MUSIC algorithm have been studied under the assumption of stationary additive Gaussian or second order model observation noise [5]. Many approaches are considered in the literature to improve the DOA estimation such as data fusion techniques [6–8]. When the array output signals are affected by impulsive noise, sudden bursts or sharp spikes [9], the above techniques become useless. Note that the duration and occurrence of the aforementioned phenomenon are arbitrary, and may originate from natural changes or jamming.

Estimation problem in presence of these types of noise has been considered in several references such as [10–19] and robust algorithms are presented. In particular, the well-known high resolution algorithms such as MUSIC and ESPRIT are based on the estimation of the desired subspace computed from the covariance matrix. To improve the robustness of these algorithms in impulsive environments, several authors propose to use a robust estimate of the covariance matrix [18,20]. These algorithms show better performance but suffer from high computational cost.

In this paper, we propose a new effective approach to overcome the problem of DOA estimation in a non-homogeneous environment. Our approach is based on data fusion technique and the well-documented performance analysis results of the considered DOA algorithm. In fact, the received array output data can be divided into segments such that one can expect these segments to not exhibit the same information quality on the DOA sources. This information can then be exploited by using data fusion concepts [6] to improve the performance of the DOA estimation.

Note that data fusion concepts are a well-known and are applied in many technological areas to improve parameter estimation. The novelty introduced in this paper can be summarized into two points: (i) a data fusion technique is implemented locally for the same data array (time data segmentation) in contrast to existing methods that use the spatial diversity (delocalized sensors) as in [8,7], and (ii) to our knowledge, it is the first time that the-

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oretical performance analysis of a DOA algorithm is used to derive a new version of the DOA algorithm under consideration.

To show the effectiveness of the proposed approach, one can consider the conventional MUSIC algorithm which is applied to each segment to provide DOA estimates. The latter are then fused on the basis of their accuracy to obtain a fused DOA using the federated fusion algorithm [21]. The DOA accuracy expression is obtained from the well-established performance analysis of DOA MUSIC-based algorithms [22–25].

The aim of this paper is to demonstrate that the proposed approach based on data fusion concepts and the well-known performance analysis can improve significantly subspace based algorithms. Herein, we limit ourselves to the general case of Gaussian environments with arbitrary bursts. Moreover, the bursts are assumed to be randomly distributed over the observed array data with unknown variances and duration.

To illustrate our claims, the performance of the proposed DOA estimation technique, referred to as FU-MUSIC, is compared with respect to the conventional MUSIC algorithm and its robust version [20] in terms of Root Mean Square Error (RMSE) and angle resolution. The obtained results are promising and show that data fusion techniques can improve efficiently the performance of the conventional MUSIC algorithm in non-homogeneous environments. Moreover, if the conventional MUSIC is substituted by a robust version of MUSIC, our approach is still effective.

This paper is organized as follows: In Section 2, the problem formulation is provided. Section 3 details our proposed approach. Some discussions on the effectiveness of our approach are made in Section 4 and Section 5 presents the results of numerical experiments supporting our claims. Finally, concluding remarks are given in Section 6.

2. Data model

Consider K narrow-band far-field signals received by a uniform linear array of L antennas ($L > K$). The array output vector $\mathbf{x}(t) = [x_1(t) \dots x_L(t)]^T$ is given by,

$$\mathbf{x}(t) = A(\theta)\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_K)]$ is the steering matrix and its ith column $(1 \le i \le K)$ is given by $\mathbf{a}(\theta_i) = [1 \;\; e^{-j\varphi_i} \;\; e^{-j2\varphi_i} \dots e^{-j(L-1)\varphi_i}]^T$, with $\varphi_i = 2\pi \frac{d}{\lambda} sin\theta_i$, λ is the propagation wavelength, d is the inter-element spacing and θ_i is the ith DOA. $\mathbf{n}(t)$ is an additive white stationary noise with zero mean and covariance matrix, $\mathbf{R_{nn}} = E\left[\mathbf{n}(t)\mathbf{n}^H(t)\right] = \sigma_n^2\mathbf{I}$ where $(.)^H$ denotes the transpose conjugate operator. $s(t) = [s_1(t) \dots s_K(t)]^T$ is the source signal vector where the sources are assumed to be uncorrelated. The current paper aims to present a new approach to improve the accuracy of DOA estimation when the observed data is affected by bursts.

3. The proposed approach

Herein, data fusion is used to improve the DOA algorithms by considering their theoretical accuracy performance analysis. In order to explain our proposed approach, we have chosen to consider the MUSIC algorithm and its well-documented accuracy performance to derive a new version.

Note that, this approach can be applied to other subspace-based DOA algorithms as long as the theoretical accuracy performance of the latter are available.

3.1. MUSIC algorithm and its performance analysis

One can compute the subspace decomposition by applying the Singular Value Decomposition (SVD) method on the observed data matrix $\mathbf{X} = [\mathbf{x}(1)\ \mathbf{x}(2) \cdots \mathbf{x}(T)]$ where T is the observation sample size. In this case, the subspace decomposition is given by

$$\mathbf{X} = \mathbf{U}\Lambda\mathbf{V}^H = [\mathbf{U}_{ss}\ \mathbf{U}_{nn}] \begin{bmatrix} \Lambda_{ss} & \mathbf{0} \\ \mathbf{0} & \Lambda_{nn} \end{bmatrix} [\mathbf{U}_{ss}\ \mathbf{U}_{nn}]^H \tag{2}$$

where Λ_{ss} is a diagonal matrix whose entries are the K largest singular values that are associated to the columns of $\mathbf{U}_{ss}$ and Λ_{nn} is a diagonal matrix whose entries are the least $L - K$ singular values that are associated to the columns of $\mathbf{U}_{nn}$. $\mathbf{U}_{ss}$ and $\mathbf{U}_{nn}$ span the signal subspace and the noise subspace, respectively.

Let $\mathbf{u}$ be any vector of the noise subspace. Thus, for any vector $\mathbf{v}$ of the signal subspace, we have $\mathbf{v}^H\mathbf{u} = 0$. In particular, this property holds for the steering vectors that span the signal subspace (i.e. $\mathbf{a}^H(\theta_k)\mathbf{u} = 0$, $k = 1, \cdots, K$).

Based on this property, the MUSIC algorithm [26] provides the estimated directions of arrival that correspond to the K local minima of the spectrum function given by

$$\mathbf{F}_{MUSIC} = \mathbf{a}^H(\theta)\mathbf{U}_{nn}\mathbf{U}_{nn}^H\mathbf{a}(\theta) \tag{3}$$

Let $\Delta\theta_k$ be the error due to the MUSIC algorithm for the estimation of θ_k such

$$\hat{\theta}_k = \theta_k + \Delta\theta_k \tag{4}$$

where $\hat{\theta}_k$ is the estimate of θ_k for $k = 1, \cdots, K$. From [22–24], and assuming that the elements of the perturbation matrix of $\mathbf{X}$ denoted $\Delta\mathbf{X}$ are uncorrelated circular random variables with equal variances σ_n^2, the performance analysis of the MUSIC algorithm provides the general expression for perturbation of DOA estimate as:

$$\Delta\theta_k = \frac{Im[\boldsymbol{\alpha}_k^H \Delta\mathbf{X}\boldsymbol{\beta}_k]}{\gamma_k} \tag{5}$$

where the parameters $\boldsymbol{\alpha}_k$, $\boldsymbol{\beta}_k$, and γ_k are given by:

$$\boldsymbol{\alpha}_k = \jmath\mathbf{U}_{nn}\mathbf{U}_{nn}^H\mathbf{a}^{(1)}(\theta_k)$$

$$\boldsymbol{\beta}_k = \mathbf{V}_{ss}\Lambda_{ss}^{-1}\mathbf{U}_{ss}^H\mathbf{a}(\theta_k) \tag{6}$$

$$\gamma_k = \mathbf{a}^{(1)}(\theta_k)^H\mathbf{U}_{nn}\mathbf{U}_{nn}^H\mathbf{a}^{(1)}(\theta_k) \tag{7}$$

and where $\mathbf{a}^{(1)}(\theta_k)$ stands for the first derivative of the steering vector $\mathbf{a}(\theta_k)$ $\left(\text{i.e. } \frac{\partial\mathbf{a}(\theta)}{\partial\theta}\right)$.

From Eq. (5) and using the above notations, a direct computation of the mean squared error is given by [23]:

$$E_{\Delta\mathbf{X}}(\Delta\theta_k)^2 = \frac{E_{\Delta\mathbf{X}}[Im(\boldsymbol{\beta}_k^H\Delta\mathbf{X}\boldsymbol{\alpha}_k)]^2}{2\gamma_k^2} \tag{8}$$

$$= \frac{\|\boldsymbol{\alpha}_k\|^2\|\boldsymbol{\beta}_k\|^2\sigma_n^2}{2\gamma_k^2} \tag{9}$$

where the notation $\|.\|$ stands for the Frobenius norm. However, the computation of the DOA estimation error (by using Eq. (9)) requires the priori knowledge of the noise power σ_n^2. The latter can be estimated as the mean of the least eigenvalues of the data covariance matrix of the observed data associated to the noise subspace.

3.2. New algorithms for DOA estimation

DOA algorithms can be improved by using data fusion technique and by considering their theoretical accuracy performance as shown below.

In the proposed methodology, the data matrix $\mathbf{X}$ is, first, divided into data segments. Then, for each data segment, the DOA