



# Monotonic sequences for image enhancement and segmentation



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## ABSTRACT

Both image enhancement and image segmentation are important pre-processing steps for various image processing fields including autonomous navigation, remote sensing, computer vision, and biomedical image analysis. Both methods have their merits and their short comings. It then becomes obvious to ask the question: is it possible to develop a new better image enhancement method which has the key elements from both segmentation and image enhancement techniques? The choice of the threshold level is a key task in image segmentation. There are other challenges of image segmentation. For example, it is very difficult to perform the image segmentation in poor data such as shadows and noise. Recently, a homothetic curves Fibonacci-based cross sections thresholding has been developed for the de-noising purposes. Is it possible to develop a new image cross sections thresholding method, which can be used for both segmentation and image enhancement purposes? This paper a) describes a unified approach for signal thresholding, b) extends cross sections concept by generating and using a new class of monotonic, piecewise linear, sequences (slowly or faster growing than Fibonacci numbers) of numbers; c) uses the extended sections concept to the image enhancement and segmentation applications. Extensive experimental evaluation demonstrates that the newly proposed monotonic sequences have great potential in image processing applications, including image segmentation and image enhancement applications. Moreover, study has shown that the generalized cross techniques are invariant under morphological transformations such as erosion, dilation, and median, able to be described analytically, can be implemented by using the look up table methods.

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## 1. Introduction

Image enhancement and image segmentation are two fundamental procedures in various image processing and computer vision applications, including autonomous navigation, remote sensing, multimedia analytics, biomedical image analysis, and astronomical images. There are extensive literature on both image enhancement and image segmentation procedures [2–9,11–17,22–41, 48–70]. The key goal of image segmentation is to split an image into its constituent components and extract important information, for example statistical, edge, contour and texture information, from an input image [1,2]. Depending on the application, the goal of segmentation can be different. In general, the segmentation is not unique and it is an ill-defined problem which makes the segmentation evaluation of a candidate algorithm difficult. “There is no unique ground-truth segmentation of an image against which the output of an algorithm may be compared” [71]. The common segmentation approaches are based on thresholding

techniques because of its simplicity, robustness, and accuracy. We can broadly categorize thresholding techniques as global thresholding, semi-thresholding, multilevel thresholding, variable thresholding, and  $(n, k, p)$ -Gray code thresholding, cross sections (threshold sets) thresholding, various histogram thresholding, supervised and unsupervised thresholding [2–9,11–17,59–70].

The cross sections based thresholding are commonly used in many computer vision and imaging processing applications, including nonlinear filtering and mathematical morphology [8,10–20,61]. In addition, the cross sections thresholding allows for transferring the basic operations of set algebra, such as Minkowski’s addition and subtraction of sets, and the dilation and erosion of sets, on the function algebra. The cross-sections are typically horizontal lines. Recently, a more general concept of homothetic curves  $g_a$ -based cross sections has been developed to generalize the traditional horizontal cross sections and define different set representations of signals and images [21] and [72]. Both these works use so called Fibonacci thresholding (based on Fibonacci numbers) levels for the denoising (median type and morphological filters) proposes. Unfortunately, many thresholding algorithms, in spite of their fame, are not able to automatically determine the required minimum number of thresholds, for a given application. Moreover, it is natural

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to ask: how to use the cross sections thresholding concept in the two-dimensional case; can we generate a new class of cross sections which are based on a new class (slowly or faster growing than Fibonacci numbers) sequence of numbers; can we tailor them to the segmentation problem, or to the image enhancement application.

Image enhancement techniques focus on improvement of the characteristic or quality of an image, and make the resulting image look “better” than the original, when compared against specific criteria [26]. Many algorithms have been proposed for image enhancement [30–50], and a comprehensive review of image enhancement techniques is presented in [29,40,41]. The following two major classifications of image enhancement techniques can be defined: the spatial domain enhancement and transform domain enhancement [25–45]. Both methods have their merits and shortcomings given differing interpretations of image data [42]. In addition, the analysis of the transform-based techniques shows that there exist common problems. Selecting parameters for the enhancement techniques is difficult, resulting in the inability of the methods to simultaneously enhance all parts of a processed image; introducing artifacts called blocking effects [48–50]. The enhancement task, however, is complicated by the lack of any general unifying theory of image enhancement, and the lack of an effective quantitative standard of image quality to act as a design criterion for an image enhancement system [26,39–42,48–52]. Furthermore, many enhancement algorithms have external parameters which are sometimes difficult to fine-tune [22,42–52]. Most of these techniques are globally dependent on the type of the input images, and treat images instead of adapting to local features within different regions [23,41].

It then becomes obvious to ask the question: Is it possible to develop a new image enhancement method, which has the key elements from both segmentation and image enhancement techniques which produce better image enhancement algorithms? This paper investigates a new general concept of weighted thresholding/representation based image enhancement. The rest of this paper is organized as follows. Section 2 introduces the concept of the monotonic sequence-based thresholding, or weighted thresholding, in signal decomposition and describes properties and different examples of the thresholding on signals and images. Section 3 describes the application of the weighted thresholding in image enhancement, which includes the histogram equalization and enhancement by the powers of the distribution function. The quantitative measure of image enhancement is calculated for the processed images. Section 4 is devoted to the weighted thresholding by piecewise linear sequences including a slowly growing stepped sequence of numbers. The thresholding by the sequence of Fibonacci numbers is also considered. Section 5 describes composition and vector median filtration of signals by Fibonacci-values signals. In Section 6, the segmentation by the morphological operations with weighted thresholding is presented. A comparison and analysis are also provided to show the new algorithm’s performance. Section 7 draws a conclusion.

## 2. Monotonic sequence-based thresholding

In this section, we present the concept of the weighted thresholding which generalizes the traditional signal decomposition by threshold sets. In such a thresholding, the binary signals of different levels are weighted by a given sequence of numbers. Properties of the weighted thresholding are described, and examples with different sequences and the corresponding signal decompositions are given.

Let  $f(x)$  be a multi-level non-negative function in the real space  $R^n$ ,  $n \geq 1$ , with integer values in the interval  $[0, L]$ ,  $L > 1$ .

We consider a sequence of numbers as a function  $k(\alpha)$  which is monotonically non-decreasing (or, non-increasing) in the set of integers  $\alpha = 1, 2, 3, \dots$

**Definition 1.** Threshold  $k(\alpha)$ -valued signals of the function  $f(x)$  at amplitude levels  $\alpha \in [0, L]$  are calculated by

$$\bar{f}_\alpha(x) = \begin{cases} k(\alpha), & \text{if } f(x) \geq \alpha, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

The representation  $f(x) \rightarrow (\bar{f}_1(x), \bar{f}_2(x), \dots, \bar{f}_L(x))$  is called the *sequence based weighted thresholding* of  $f(x)$ , or, simply the *sequence thresholding*.

### 2.1. Properties of sequence thresholding

1. If the sequence  $k(\alpha)$  is monotonically decreasing, then

$$\bar{f}_\alpha(x) \geq \bar{f}_\beta(x), \quad \text{when } \alpha < \beta. \quad (2)$$

2. The function  $f(x)$  can be reconstructed by its family of thresholded  $k(\alpha)$ -valued signals as

$$f(x) = \sup\{\alpha; \bar{f}_\alpha(x) \neq 0\} = \sum_{\alpha=1}^L \frac{\bar{f}_\alpha(x)}{k(\alpha)}, \quad x \in R^n. \quad (3)$$

3. The following function can uniquely be assigned to the function  $f(x)$ :

$$\mathcal{K}_f(x) = \sum_{\alpha=1}^m \bar{f}_\alpha(x), \quad x \in R^n, \quad (4)$$

where  $m = f(x) \leq L$ . We call this transformation the *distribution operator*.

The distribution operator  $\mathcal{K}: f(x) \rightarrow \mathcal{K}_f(x)$ , keeps all straight (smooth) parts in the graph of  $f$ , increasing its peaks.  $\mathcal{K}_f(x) \geq f(x)$  (if  $k(\alpha) \geq 1$ ), and the equality holds in the case when all values of  $k(\alpha)$  equal 1.

4. When  $k(\alpha) \equiv 1$ , the 1-valued signal  $\bar{f}_\alpha(x)$  equals the binary signal  $f_\alpha(x)$  at level  $\alpha$ , which is defined as  $f_\alpha(x) = 1$  if  $f(x) \geq \alpha$ , and  $f_\alpha(x) = 0$  otherwise. It is well known [17] that every u.s.c. function  $f(x)$  can be reconstructed uniquely by its horizontal cross sections, or the family of the binary signals  $\{f_\alpha(x); \alpha \in (-\infty, +\infty)\}$ , due to the following threshold decomposition:

$$f(x) = \sup\{\alpha; f_\alpha(x) = 1\} = \sum_{\alpha} f_\alpha(x). \quad (5)$$

5. The mapping  $f(x) \rightarrow \mathcal{K}_f(x)$  is invertible and the inversion has a simple form, even in the case when the mapping is complex. It means, that in many cases, one can replace complicated nonlinear operations by means of the simple summing (in Boolean logic) of the binary signals. This property follows directly from Eqs. (3) and (4).

6. If the signal  $f(x)$  has the equal binary signals, it can be reduced to the  $k$ -weighted thresholding. Such thresholding is called the *base* or *canonical representation* [43].

**Example 1.** Let  $f(x)$  be the signal  $\{0, 1, 5, 3, 3, 8, 0\}$  with the maximal level 8. This function  $f(x)$  can be described by two forms, the direct representation and base representation, as follows:

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