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On the realization of common matrix classifier using covariance tensors

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ABSTRACT

Due to the growing interest in image classifiers, the concept of native two dimensional (2-D) classifiers continues to attract researchers in the field of pattern recognition. In most cases, the 2-D extension of a regular 1-D classifier is straightforward. Following the construction methodology of the Common Matrix Approach (CMA), its relation to the eigen-matrices of the covariance tensor is illustrated. The proposed methodology presents an alternative point of view to the classical CMA implementation that depends on Gram–Schmidt orthogonalization. Therefore a 2-D approach which is the counterpart of CVA implemented with covariance matrix is developed in this paper.

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1. Introduction

The amount of 2-D data and images continuously increase in digital form. An immediate application of such data is detection and classification. The conventional approach to classification in images was the adaptation of well known statistical or neural classifiers into the 2-D data. When the classifier requires subspace projections or transformations, these operations were usually performed along the rows and the columns of the image in the form of consecutive 1-D processes. Although such operations handle most of the classification problems, they may lack the exact data dependencies in two dimensions. In such cases, construction of a *native* 2-D classifier with the desired properties may become essential. The subspace classifier, known as the Common Vector Approach (CVA) [1–5], has the problem of extension difficulty to higher dimensions.

In the literature, the classical CVA was defined for 1-D time signals using two methods: 1) via Gram–Schmidt orthogonalization, 2) via eigen-decomposition of covariance matrices. Since CVA construction idea originally comes from the orthogonalization of data, its 2-D extension was only applied through Gram–Schmidt orthogonalization. In that aspect, generalization of eigen-analysis to higher dimensions was non-trivial, so never utilized in the literature. In this work, the extension of CVA to 2-D, which is called CMA, is constructed using a tensor-style eigenvalue–eigenmatrix framework, and its analogy to the classical orthogonalization-based construction of CMA is illustrated. Specifically, fourth-order covariance tensors are constructed in order to examine the spatial relationships between the image pixels effectively. These covariance tensors for each class are obtained from second-order tensors (matrices) of that class. The algebraic equivalence of tensor based eigen-analysis and Gram–Schmidt orthogonalization is mathematically achieved and numerically verified over a Face Recognition case study that utilizes CVA (or, CMA, thereof) as a classifier with the above two implementations. Although the tensor-style eigen-analysis does "not" provide a certain computational advantage over the orthogonalization process (with essentially an identical complexity), the presented equivalence provides a mathematical insight to the idea of CVA through small eigenvalues and corresponding eigen(vectors/matrices). Besides, it opens the possibility to further investigate the classification via the *common* vectors even for higher dimensional signals.

The organization of the paper is as follows: In Section 2, the covariance-based and subspace classifiers are first briefly mentioned to motivate the implementation of CVA. Extensions of PCA and LDA to 2-D or higher dimensions using the tensor notation are mentioned in this section. In Section 3, the immediate 2-D version of CVA (namely, the Common Matrix Approach – CMA) is constructed using the initial idea of producing orthonormal difference matrices and the corresponding subspaces spanned by them. In Section 4, the main contribution of this work is presented where the CMA construction is performed by the eigenmatrix determination of covariance tensors. The analogy and equivalence between the constructions of Section 3 and those of Section 4 are described

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in Section 5, where the equality of the final projection operators are proved for both zero eigenvalues and non-zero eigenvalues of the covariance tensor. A separate analysis for the zero and the non-zero eigenvalues is also carried out due to the inherent construction and the working idea of CVA. In the same section, the described analogy is illustrated using a simple numerical example. In Section 5, both constructions of CMA method are also tested on a real life classification application of face recognition and the equivalences of both constructions are verified. The discussions and conclusions are given in Section 6.

2. Literature background

Image classification has been an active research field since decades. Several classification methods had been proposed and applied to specific problems such as surveillance, diagnosis, face recognition, etc. Most classical algorithms, such as Principal Component Analysis (PCA) [6] and Linear Discriminant Analysis (LDA) [7], treat an input image as a 1-D vector rendered in raster scan [8,9]. However, some recent works have started to consider an object as a two dimensional matrix for unsupervised learning. Recently, multilinear algebra, the algebra of higher-order tensors, was applied for analyzing the structure of image ensembles [10,11]. Being the natural generalization of matrices, tensors define multilinear operators over a set of vector spaces [11]. Hence, tensor analysis is a unifying mathematical framework suitable for a variety of visual problems. For example, a renewal of interest in the use of higher-order statistics (HOS) has been clearly visible especially on tensors in the last few years [12].

Being a very popular image classification problem, face recognition is selected as an example application in this work. Most face recognition techniques focus on important portions of face to perform expression-independent face recognition. In particular, PCA and its refinement, Independent Component Analysis (ICA) has been extensively used in facial image recognition [7,13,14]. The conventional eigenfaces technique [6,9] works well when person identity is the only varying parameter. If other factors, such as illumination, posing, and expression are also permitted to vary, the eigenfaces technique experiences accuracy problems. ICA can also be made by means of simultaneous third-order tensor diagonalization (STOTD) approach which is mainly similar to the well known and efficient Joint Approximate Diagonalization of Eigenmatrices (JADE) algorithm [15,16]. PCA-based ICA routines [15] can also be mentioned in the area of such modifications. Similarly, 2D-PCA and Kernel PCA (KPCA) are all modifications or extensions of PCA to address higher order statistical dependencies [17,18]. The incremental learning of bidirectional principal component analysis (IBDPCA) algorithm [19] was presented by a matricization of third-order tensor for on-line training. If all training data is not given in advance, and new training samples arrive at any time, the IBDPCA algorithm overcomes the shortcomings of BDPCA [19].

Another linear technique that has been successfully applied to face recognition is LDA [20]. The Fisher's LDA (FLDA) [21] method further overcomes the limitations of eigenfaces method. Another extension to LDA was proposed by Kong et al. [22], where the small sample size problem in LDA was resolved by utilizing a Two Dimensional Fisher Discriminant Analysis (2DFDA) algorithm [22]. Similarly, Deng et al. [23] have proposed two novel tensor subspace learning algorithms called TensorPCA and TensorLDA. The "tensorfaces" method strongly resembles the construction methodology utilized in this work. This method was proposed to make different representations of faces and it has several advantages over conventional eigenfaces [24]. Recently, Sp-Tensor method [25] was developed to extend tensorface by applying the sub-pattern technique. The advantages of the Sp-Tensor method over tensorface are not only a portion of spatial structure and local information of facial images is preserved but also the computational complexity dramatically reduces [25]. A novel efficient appearance-based face recognition method called Tensor Subspace Regression (TSR) has been suggested [26] to overcome the shortcomings of Tensor Subspace Analysis (TSA) since it is time consuming and needs to solve a series of eigen-problems. The difference between TSR and TSA is that the facial subspace learning problem was implemented in a regression framework which avoids the high computational eigenstep [26].

Similar to matrix eigenvalue decomposition (EVD), tensor higher-order EVD (HOEVD) has been used for reconstructing the pathways in cellular systems [8]. A novel feature fusion method called Multiple Component Analysis (MCA) has been proposed by constructing a higher-order covariance tensor [27]. In MCA, the orthogonal subspaces corresponding to each feature set have been learnt through tensor Singular Value Decomposition (SVD) that couples dimension reduction and feature fusion together [27]. As another utilization of the covariance tensor concept, a special (index-free) tensor formalism (in which, quadri-covariance tensor and its eigenmatrices are natural 4th-order generalizations of 2nd-order covariance) has been built to express 4th-order multivariate statistics [12]. A multimodal recognition based on feature fusion and "curve tensor approach" was proposed to address the small sample size problem in biometric recognition [28]. Generally 2nd or higher order tensors have been explored for the aim of encoding an object and the characteristics of the higher-order-tensor-based discriminant analysis has been investigated in [29]. The next transform method is the Higher Order SVD which was successfully applied to human face recognition [30] and human motion analysis problems [24,31–33]. Finally, Lathauwer and Castaing [34] have proposed new deterministic tensor-based techniques for the blind separation of a mixture of Direct Sequence–Code Division Multiple Access (DS–CDMA) signals [34].

CVA was successfully used in speech recognition [2–4], speaker recognition [1], image recognition [35,36] and motor fault detection [5] problems. The work presented here is considered as an extension of CVA to a higher dimension, producing the Common Matrix Approach (CMA). Due to the similarity of the implementation between CVA and PCA or LDA, the higher dimension extensions have a similar flavor. Previous studies on CMA have been realized by finding the orthonormal difference matrices and constructing difference subspaces for each image class [37]. In this work, CMA is implemented with a completely new procedure, and its verification with the classical CMA construction technique is performed. The CMA method is believed to have a potential in image processing and classification.

The regular CVA can be explained and implemented with two different approaches:

1. Subspace construction via orthonormal difference vectors, and
2. Eigenvector construction using covariance matrices for each subspace.

Until now, CMA could only be performed using orthonormal difference matrices (like the orthonormal difference vectors of CVA, corresponding to the approach numbered 1 in above). CMA has an advantage in terms of computational complexity because the covariance matrices obtained from image vectors cannot be easily processed. In this work, it is shown that the second approach can be extended into 2-D space and thus CMA can be implemented using covariance tensors, producing exactly the same classifier results. Showing the existence of CMA implemented with 2-D counterpart of second approach would be incomplete without showing its relation to the covariance tensor approach. Using the application of face recognition, it was illustrated that CMA is com-

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