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Blind extraction of chaotic sources from mixtures with stochastic signals based on recurrence quantification analysis

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ABSTRACT

This work aims to present a new method to perform blind extraction of chaotic deterministic sources mixed with stochastic signals. This technique employs the recurrence quantification analysis (RQA), a tool commonly used to study dynamical systems, to obtain the separating system that recovers the deterministic source. The method is applied to invertible and underdetermined mixture models considering different stochastic sources and different RQA measures. A brief discussion about the influence of recurrence plot parameters on the robustness of the proposal is also provided and illustrated by a set of representative simulations.

1. Introduction

Dynamical systems can be described in terms of a state mapping commonly defined by a set of differential equations. When this mapping is composed of nonlinear functions, a very rich dynamical scenario can occur, which includes convergence to fixed points, existence of limit-cycles, quasi-periodicity and chaos. Indeed, chaotic oscillations are present in many physical systems (e.g. biological, mechanical and electronic), which is justified by the relevance to the study of natural phenomena of nonlinear processes like cooperation, competition, saturation and hysteresis, just to cite a few. Chaotic behavior is associated with features as aperiodicity, broadband spectrum and sensitivity to initial conditions, aspects that can be easily confused with characteristics of random processes [1,2].

In fact, distinguishing chaotic from random signals is a far from trivial task, especially when experimental time series immersed in noise are considered [2–5]. The most common approach is to evaluate the Kolmogorov–Sinai (KS) entropy by calculating its lower bound given by the correlation entropy (K2) [6]. In general, this quantity is zero for periodic signals, finite and positive for chaotic processes and tends to infinite for random signals [2,7], although some stochastic processes characterized by a power law spectrum can be cited as exceptions, providing a finite and positive value for the K2 entropy [2]. Moreover, invariant measures that characterize chaotic dynamical systems (as the K2 entropy, Lyapunov exponents, correlation dimensions, among others) are strongly affected by noise in practice, which makes their calculation from experimental time series unstable or unreliable [1]. In this case, it is certainly of great use to employ a preprocessing stage in order to enhance, for instance, the deterministic features of the signal. Unfortunately, the filtering process based on the classical Fourier approach can cause the loss of relevant information [1,8–10], since both signals (chaotic and random) have a broadband spectrum. In this context, the challenging problem of denoising chaotic time series has been addressed in several works [11–15]. Generally, these methods constrain the reconstructed state vector to fall onto geometrical objects that are locally linear (or higher-order polynomial maps) [1,10], assuming that the deterministic component lies on a smooth submanifold (see [9,10] for interesting reviews), which makes possible to achieve the trajectory generated by the dynamics, reducing noise by an iterative process.

From a theoretical standpoint, if more information is available (which, in this work, means that more than one mixture of the chaotic and noise sources can be available), denoising chaotic time series can be treated within the framework of the blind source extraction (BSE) problem, as it is concerned with recovering a specific set of signals of interest – usually the deterministic signals – from versions in which they are mixed with stochastic sources. A natural possibility to solve this problem would be to employ a classical blind separation approach such as the well-established independent component analysis (ICA) [16,17], although it would not be capable of exploring the peculiar features of the problem, particularly the fact that some signals are generated by a deterministic dynamical system. As a matter of fact, this is an instance in which

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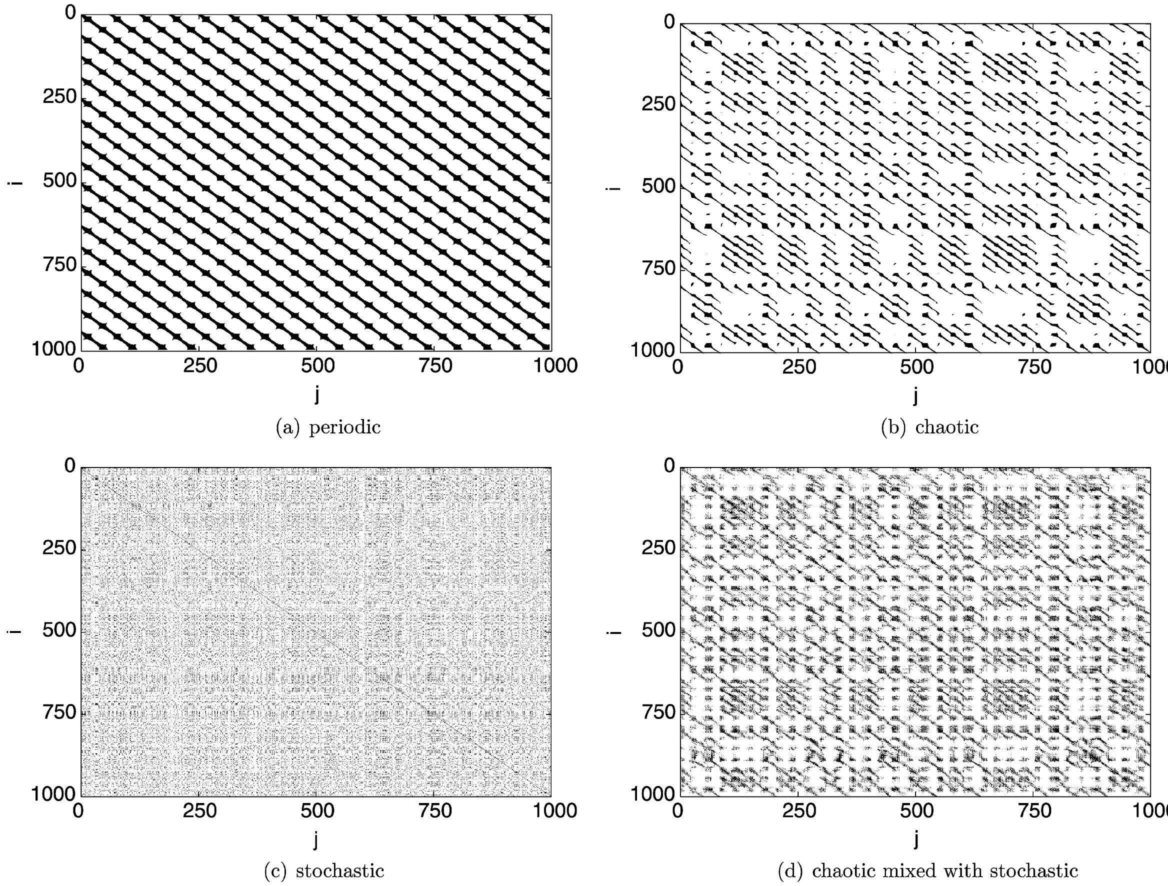

Fig. 1. Panels (a), (b), (c), (d), show, respectively, the recurrence plots ($N = 1000$ samples, $d_e = 3$, $\tau = 3$, $\varepsilon = 0.5$) from a periodic oscillation ($\sin(10t)$), a chaotic Lorenz time series, a gaussian random source (zero mean and unitary variance) and a mixture of random and chaotic sources.

a priori information about the sources is available, which is always something that widens the applicability of blind signal processing (research areas such as sparse component analysis attest this fact [18]).

In this work, a method for solving the BSE problem when chaotic and stochastic processes are mixed is presented. The technique explores the dynamic features underlying the generation of the chaotic sources to recover a signal that is "as deterministic as possible". The solution employs a recurrence plot, a classical tool for nonlinear analysis of dynamical systems [19,20], to build score functions based on classical estimators given by recurrence quantification analysis (RQA) [21]. These score functions are used to adapt linear separating systems under different signal and mixture models (invertible and underdetermined), and a comparison with a classical ICA methodology is established.

This work is organized as it follows: in Section 2, a brief introduction to chaotic signals and RQA is given. Section 3 presents the BSE problem and its relation to the proposed approach to extract deterministic sources. Section 4 is dedicated to showing the results obtained for a perfect invertible scenario (in which a full rank mixing matrix is considered), to analyzing the role of recurrence plot parameters in the extraction procedure, and finally, to presenting the performance of the method in the underdetermined case (in which there are more sources than mixtures). Section 5 presents a discussion about the contributions and perspectives of application of the proposed method in view of what has already been exposed in the literature. The idea of extraction of chaotic sources using RQA was introduced by the present authors in a previous work ([22]) and tested for a limited set of simulations. The present work extends the proposal by taking in account different stochastic sources and mixing models considering three classical RQA measures, and also by analyzing the role of recurrence plot parameters on the extraction procedure.

2. Chaotic signals and generation of recurrence plots

In formal terms, a chaotic signal is defined as a continuous-valued signal with finite and positive entropy rate and infinite redundancy rate [7]. For our purposes, a chaotic signal should be simply understood as one generated by a chaotic system, which means that its properties are defined by the dynamics that generates it and the behavior of its trajectories in the phase space. In order to reconstruct the underlying attractor (the solution of the dynamical equations in the phase space) from a single observed signal (that is, from a single state variable) one can apply the Takens' embedding theorem [1], defining a state vector $\mathbf{x}(k)$ such that:

$$\mathbf{x}(k) = \begin{bmatrix} x(k) & x(k-\tau) & \ldots & x\big(k-(d_e-1)\tau\big) \end{bmatrix} \quad (1)$$

where d_e represents the embedding dimension – defined as the number of coordinates that unfolds the attractor – and τ represents the delay between samples. Even though this trajectory may not be exactly the same as that generated by the system, it will be topologically equivalent thereto [1].

After the reconstruction, it is possible to characterize the attractor with the aid of its revisited states, which can be done with a recurrence plot, a useful graphical tool for nonlinear analysis of dynamical systems first proposed in [19]. Using the reconstructed state $\mathbf{x}(k)$, the recurrence map will be represented by an $N \times N$ matrix, where the element (i, j) will be a black dot whenever $\mathbf{x}(i)$ is sufficiently close to $\mathbf{x}(j)$, i.e., whenever $\|\mathbf{x}(i) - \mathbf{x}(j)\| < \varepsilon$.

The characterization and applicability of recurrence plots becomes clear by comparing maps obtained from signals of different