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A Gaussian mixture Bernoulli filter for extended target tracking with application to an ultra-wideband localization system ☆

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ABSTRACT

This paper presents a Gaussian mixture (GM) implementation of the Bernoulli filter for extended target tracking, which we call the extended target GM Bernoulli (ET-GM-Ber) filter. Closed form expressions for the ET-GM-Ber filter recursions are obtained. A clustering step is integrated into the measurement update stage in order to have a computationally tractable filter. Performance of the proposed filter is tested both on the simulated data and experimental data collected using an ultra-wideband (UWB) localization system. Simulations and experimental results demonstrate the accurate and effective performance of the proposed filter.

1. Introduction

Target tracking can be considered as maintaining estimates of the current state of the target by processing the measurements collected by a sensor [1]. The classical target tracking approach, which assumes that each target gives rise to at most one measurement per time step, is a well-established research area. However, with the advent of modern, high resolution sensors such as ultra-wideband (UWB) sensors and automotive radars, targets have to be treated as extended targets which can produce multiple measurements per time step. Considering extended target tracking framework, we are not interested in estimating the position of each point that generates the individual detections since these point positions in general fluctuate depending on the time-varying sensor-to-target geometry. In extended target tracking, we are interested in position estimation of the target as a whole. Recent advances in electronic industry and computation speeds have made extended target tracking feasible and this has attracted researchers' attention [2,3]. Concerning the extended target tracking, several different applications are studied in the literature [4–7].

Random finite set (RFS) based tracking approach introduced by Mahler has emerged as a promising alternative to the traditional association-based methods [8]. As a mathematically principled and theoretically optimal framework, the RFS paradigm has attracted considerable research interest during the last decade [9–11,8,12,13]. Different approximations have been developed recently for computationally tractable solutions: the Bernoulli filter [8,12,13], the probability hypothesis density (PHD) filter [14,15] the cardinalized PHD (CPHD) filter [16,17], the multi-Bernoulli filter [8,18] and the labeled versions of corresponding RFS filters [19]. The PHD filters have been successfully used in many different applications [20–25]. PHD-based approaches have also been developed for extended target tracking [26–28,2,29].

The Bernoulli filter is the exact Bayes filter which propagates the parameters of a Bernoulli RFS for a single dynamic system which can randomly switch on and off (i.e. target birth/death) [30, 9,11]. Compared to the works done in extended target tracking using the PHD-like filters, other than the works of Ristic et al. in [30, 7], there exists almost no work on extended target tracking using the Bernoulli filter. Only, in a very recently published article, authors propose a filter for multiple extended target tracking based on labeled RFS [31]. In this work, we present a Gaussian Mixture (GM) implementation of the extended target Bernoulli filter which can jointly detect and track a single target in the presence of detection uncertainty, target-measurement rate uncertainty, noise and false alarms. GM implementation of the RFS-based filters has been widely used, as it provides a closed form solution to filter recursions under linear Gaussian target dynamics and measurement models. Furthermore, one of the main benefits of GM implementation is that its state estimates are obtained from the posterior intensity in an easy and efficient manner [15]. In the proposed filter, the clustering approach in [27] is adapted into the extended target Bernoulli filter in order to have a tractable implementation. The output of a clustering step is used to estimate the time

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varying number of scatters of the extended target. Performance of the proposed filter, which is called as the extended target Gaussian Mixture Bernoulli filter (ET-GM-Ber), is compared with the point-target GM Bernoulli (GM-Ber) and extended target Gaussian Mixture PHD (ET-GM-PHD) filters [30].

The novelty of this work is two-fold:

- We present the GM implementation of the Bernoulli filter, which assumes binomial RFS model of target-originated measurements, for extended target tracking.
- We demonstrate the performance of the proposed filter using the measurements of an UWB sensor network.

In the UWB sensor network experiments, we use the Ubisense UWB Real-Time Location Systems (UWB-RTLS) [32]. UWB has emerged as a very promising technology having certain advantages such as high resolution, robustness to multi-paths, low power and small size [33–35]. Thus, UWB sensor systems can be considered as a leading technology for many applications [36,37,22,38] and they are particularly well-suited for in-door positioning [33,39,40,38]. Target tracking with UWB sensors under point-target assumptions has been considered so far [41–44]. However, there exists no work employing the concept of extended target using UWB sensors. Therefore, our work presents an extended target tracking application with UWB sensors for the first time. We show that extended target tracking with UWB technology is practical.

The paper is organized as follows. Section 2 presents the RFS based problem formulation and Bernoulli filter for extended target tracking. Section 3 gives the details of the ET-GM-Ber filter implementation. Simulation results are presented in Section 4 and experiment results are given in Section 5. Finally, Section 6 provides conclusions and possible future research ideas.

2. Problem formulation

In this section, we provide some key points of the RFS framework to be used in the proposed ET-GM-Ber filter.

2.1. Random finite sets (RFS) based filtering

An RFS can be considered as a random variable whose values are unordered finite sets. Thus, the cardinality of a RFS $\mathbf{X}$ is randomly distributed and modeled by a discrete probability distribution $\rho(n) = P(|\mathbf{X}| = n)$, where $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \mathcal{X}$, $n \in \mathbf{N}_0$ and $\mathbf{N}_0$ denotes the set of natural numbers including zero. A RFS $\mathbf{X}$ is completely characterized by its cardinality distribution $\rho(n)$ and a family of symmetric joint distributions $p_n(\mathbf{x}_1, ..., \mathbf{x}_n)$. By employing the Mahler's finite set statistics (FISST) [8], $f(X)$ represents the FISST probability density function (FISST PDF) [8,30]. This PDF is uniquely specified by $\rho(n)$ and $p_n(\mathbf{x}_1, ..., \mathbf{x}_n)$ as follows [30]:

$$f(\{\mathbf{x}_1, ..., \mathbf{x}_n\}) = n!\rho(n)p_n(\mathbf{x}_1, ..., \mathbf{x}_n) \; . \tag{1}$$

The set integral is defined as [8]:

$$\int f(\mathbf{X})\delta\mathbf{X} = \sum_{n=1}^{\infty} \frac{1}{n!} \int f(\{\mathbf{x}_1, ..., \mathbf{x}_n\})d\mathbf{x}_1, ..., \mathbf{x}_n \; . \tag{2}$$

As in classical PDF, $f(\mathbf{X})$ integrates to one since it is also a PDF defined in FISST framework.

In the RFS framework, both targets and measurements take values as unordered finite sets. Thus, the target state and measurement RFS can be formulated as:

$$\mathbf{X}_k = \{\mathbf{x}_k^i, ..., \mathbf{x}_k^{M(k)}\} \in \mathbb{F}(\mathcal{X}) \; , \tag{3}$$

$$\mathbf{Z}_k = \{\mathbf{z}_k^i, ..., \mathbf{z}_k^{N(k)}\} \in \mathbb{F}(\mathcal{Z}) \; , \tag{4}$$

where $M(k)$ and $N(k)$ correspond to the number of targets and measurements respectively at time k, $\mathbb{F}(\mathcal{X})$ and $\mathbb{F}(\mathcal{Z})$ are the set of all possible finite sets of state space $\mathcal{X}$ and measurement space $\mathcal{Z}$ correspondingly. The multi-target state evolution is described by a first-order Markov process with transitional density $\phi_{k|k-1}(\mathbf{X}_k|\mathbf{X}_{k-1})$. The likelihood function of $\mathbf{Z}_k$ is represented by $\varphi_k(\mathbf{Z}_k|\mathbf{X}_k)$. Furthermore, the sequence of measurements up to time k is denoted by $\mathbf{Z}_{1:k}$. Suppose that at time $k-1$, the posterior FISST PDF of multi-target state $f_{k-1|k-1}(\mathbf{X}_{k-1}|\mathbf{Z}_{1:k-1})$ is known. Then, the predicted and updated multi-target posterior densities are given by [8]:

$$f_{k|k-1}(\mathbf{X}_k|\mathbf{Z}_{1:k-1}) = \int \phi_{k|k-1}(\mathbf{X}_k|\mathbf{X}_{k-1}) f_{k-1|k-1}(\mathbf{X}_{k-1}|\mathbf{Z}_{1:k-1})\delta\mathbf{X}_{k-1} \; , \tag{5}$$

$$f_{k|k}(\mathbf{X}_k|\mathbf{Z}_{1:k}) = \frac{\varphi_k(\mathbf{Z}_k|\mathbf{X}_k) f_{k|k-1}(\mathbf{X}_k|\mathbf{Z}_{1:k-1})}{\int \varphi_k(\mathbf{Z}_k|\mathbf{X}_k) f_{k|k-1}(\mathbf{X}_k|\mathbf{Z}_{1:k-1})\delta\mathbf{X}_k} \; . \tag{6}$$

Next, we define some common RFSs relevant to our work [30].

2.1.1. Bernoulli RFS

The cardinality of a Bernoulli RFS is Bernoulli distributed. Therefore, the Bernoulli RFS can either be empty set with probability $1-q$ or it can have one element with probability q distributed over the state space $\mathcal{X}$ with PDF $p(\mathbf{x})$. The FISST PDF of a Bernoulli RFS $\mathbf{X}$ is given by:

$$f(\mathbf{X}) = \begin{cases} 1-q & \text{if } \mathbf{X} = \emptyset \\ q \cdot p(\mathbf{x}) & \text{if } \mathbf{X} = \{\mathbf{x}\} \end{cases} \tag{7}$$

2.1.2. Binomial RFS

A Binomial RFS $\mathbf{X}$ is an independent identically distributed (IID) cluster RFS. Its cardinality distribution is a binomial distribution with parameters L (number of binary experiments) and q (the probability of success of each of the experiments):

$$\rho(n) = \binom{L}{n} q^n (1-q)^{L-n}, \;\; n = 0, 1, 2, ..., L. \tag{8}$$

Its FISST PDF is defined as:

$$f(\mathbf{X}) = \frac{L!}{(L-|\mathbf{X}|)!} q^{|\mathbf{X}|} (1-q)^{L-|\mathbf{X}|} \prod_{\mathbf{x}\in\mathbf{X}} p(\mathbf{x}) \; . \tag{9}$$

When $L = 1$, the Binomial RFS reduces to the Bernoulli RFS.

2.1.3. Poisson RFS

A Poisson RFS $\mathbf{X}$ is a kind of IID cluster RFS and its cardinality is Poisson distributed such that:

$$\rho(n) = \frac{e^{-\lambda}\lambda^n}{n!}, \;\; n = 0, 1, 2, ... \tag{10}$$

Its FISST PDF is defined as:

$$f(\mathbf{X}) = e^{-\lambda} \prod_{\mathbf{x}\in\mathbf{X}} \lambda p(\mathbf{x}) \; . \tag{11}$$

2.2. Bernoulli filter for an extended target

In this section, we summarize the key points of the Bernoulli filter for extended target tracking [8,30]. Bernoulli filter is also a Bayesian optimal recursive filter, thus it involves two steps: prediction and update. As usual, prediction is achieved based on the state transition density which describes the object interim motion between measurements according to Eq. (5). And then, update step is applied based on the likelihood function which describes the sensor model according to Eq. (6).