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Impulsive noise suppression in the case of frequency estimation by exploring signal sparsity

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ABSTRACT

The frequency estimation problem is addressed in this work in the presence of impulsive noise. Two typical scenarios are considered; that is, the received data are assumed to be uniformly sampled, i.e., without data missing for the first case and data are randomly missed for the second case. The main objective of this work is to explore the signal sparsity in the frequency domain to perform frequency estimation under the impulsive noise. Therefore, to that end, a DFT-like matrix is created in which the frequency sparsity is provided. The missing measurements are modeled by a sparse representation as well, where missing samples are set to be zeros. Based on this model, the missing pattern represented by a vector is indeed sparse since it only contains zeros and ones. The impulsive noise is remodeled as a superposition of a unknown sparse vector and a Gaussian vector because of the impulsive nature of noise. By utilizing the sparse property of the vector, the impulsive noise can be treated as a unknown parameter and hence it can be canceled efficiently. By exploring the sparsity obtained, therefore, a joint estimation method is devised under optimization framework. It renders one to simultaneously estimate the frequency, noise, and the missing pattern. Numerical studies and an application to speech denoising indicate that the joint estimation method always offers precise and consistent performance when compared to the non-joint estimation approach.

1. Introduction

Frequency estimation is a fundamental problem in estimation theory and it has found a wide range of applications in communications, radar, and speech [1,2]. In the past decades, there has been considerable interest in this research topic and a variety of frequency estimation methods have been developed. The traditional methods of frequency estimation [3–6] are usually developed under the assumption that the additive noise is Gaussian distribution, but, for example, actual wireless transmission system often exists for some non-Gaussian noise, which has a significant peak pulse waveform and thicker tail probability density function, see [7]. This type of noise is impulsive noise and it can be perfectly modeled by α-stable distribution [8,9] in which α is the characteristic exponent and it will be defined later. In fact, it is a generalized version of Gaussian noise. The traditional methods designed for Gaussian noise cannot function well under the impulsive noise. Therefore, new approaches are needed to handle this type of noise. When it comes to impulsive noise reduction, the most popular choice of dealing with it is to utilize ℓ_p-norm on data fidelity since $\|x\|_p$ exists when $p < \alpha$ [10,11]. It follows from [12] that the M-estimate can be adopted for handling impulsive noise as well.

The second issue often encountered in practice is missing measurements due to sensors' failure, data transmission loss, and other unknown reasons. The classic approaches designed with uniform samples in mind render themselves ineffective under this case. In [13], a gapped-data amplitude and phase estimation (GAPE) algorithm is developed. However, it cannot work under the arbitrary missing pattern case. The autoregressive moving average (ARMA) model developed in [14] is utilized to fit the least square error to estimate frequency. Unfortunately, the local minimas prevent the algorithm to obtain the optimal solutions. In 2009, the authors in [15] proposed an iterative adaptive approach (IAA) to estimate frequency, where a weighted least squares concept is adopted. The robust modeling technique was also studied to enhance the system robustness in the case of disturbances in [16].

To handle those issues, namely missing measurements and impulsive noise, in this paper, a joint estimation approach is designed to simultaneously recover the frequency and missing pattern under

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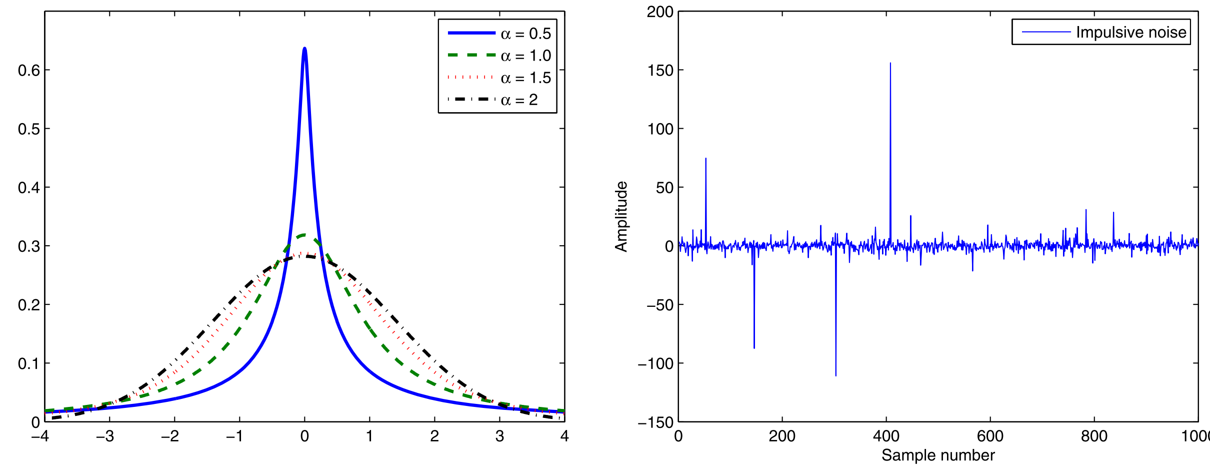

(a) PDFs of α-stable distribution with $\gamma = 1, \beta = 0, a = 0$

(b) Realization of α-stable distribution with $\alpha = 1.5, \gamma = 2, \beta = 0, a = 0$

Fig. 1. The α-stable distribution.

the impulsive noise condition. The primary objective of this work focuses on how to utilize signal sparse property [17] to achieve that goal. The frequency sparsity can be easily spotted by transforming the signal into frequency domain. Due to the impulsiveness property of the noise, the nearly-sparsity is also observed in time domain since it only has few big values and lots of small values. To find a sparse representation of missing pattern, a sparse vector that consists of only zeros and ones is introduced to remodel the missing measurements problem. With the aid of the reformulations performed, a joint estimation approach under an optimization framework is readily developed to perform frequency and missing pattern estimation while suppressing the impulsive noise. To efficiently solve this optimization problem, a two-step iterative approach is proposed as well. To demonstrate the performance of the proposed joint estimation approach, an application of speech denoising is considered. When people converse online, the conversation is usually interrupted by the keyboard knocking, which is treated as the unwanted noise. Due to the fact that the duration of keyboard clicking is usually shorter and stronger when compared to the speech signal, this noise can be represented by the impulsive noise. To create a better conversation environment, the noise is desired to be suppressed. For more details, the interested readers are referred to [18–20].

The rest of the paper is organized as follows: In Section 2, the α-stable distribution is introduced that provides a foundation to model the impulsive noise. A joint estimation approach is presented to estimate the frequency, the impulsive noise in Section 3, and to estimate the missing pattern in Section 4, respectively. In Section 5, a speech denoising problem is presented and the joint estimation approach is utilized to reduce the noise. In Section 6, numerical studies for the proposed approach are presented. Finally, this paper concludes with a brief summary in Section 7.

2. Impulsive noise

2.1. α-Stable distribution

The generalized central limit theorem states that if the sum of independent and identically distributed (i.i.d.) random variables with or without finite variance converges to a distribution by increasing the number of variables, the limit distribution must belong to the family of stable laws, see [10]. The main difference between the stable and Gaussian distributions is that the tails of the stable density are heavier than that of the Gaussian density. However, in general, there is no closed-form expression for the probability density function (PDF) of stable distributions. The most convenient way to define them is to use the characteristic function. That is [10]

$$\varphi(t) = \exp\{jat - |\gamma t|^{\alpha}[1 + j\beta \mathrm{sign}(t)\omega(t,\alpha)]\}, \tag{1}$$

where

$$\omega(t,\alpha) = \begin{cases} -\tan\frac{\pi\alpha}{2}, & \text{if } \alpha \neq 1 \\ 2/\pi \log|t|, & \text{if } \alpha = 1 \end{cases}$$

$$\mathrm{sign}(t) = \begin{cases} 1, & \text{if } t > 0 \\ 0, & \text{if } t = 0 \\ -1, & \text{if } t < 0 \end{cases}$$

In (1), the meanings of the four parameters α, γ, β, a are given as follows:

(i) α $(0 < \alpha \leq 2)$ is the characteristic exponent. It is the most important parameter, which determines the shape of the distribution and controls the heaviness of the tails of the density function. The heavier of the tails is, the more severe the impulsiveness is.

(ii) γ $(\gamma > 0)$ is the dispersion parameter, which determines the spread of the density and acts in a similar way to the variance of the Gaussian density.

(iii) β $(-1 \leq \beta \leq 1)$ is the symmetry parameter and $\beta = 0$ corresponds to the symmetric α-stable (SαS) distribution, i.e., symmetric about a.

(iv) a $(-\infty < a < \infty)$ is the location parameter, which is the mean when $1 < a \leq 2$ and the median when $0 < a < 1$ for SαS distributions.

It is interesting to note that the p*th*-moment with $p < \alpha$ exists, which will be used later in algorithm development. Due to the impulsive nature of the noise, the algorithms designed for the Gaussian distributed noise will not work in this case. However, the ℓ_p-norm is widely used for this kind of noise. However, one exciting property we notice is that due to impulsive nature, this type of signal can be seen as a nearly-sparse signal. This property can be more clearly observed in Fig. 1 in which the impulsive noise usually has a few big spikes and lots of small values. This observation turns out to be our benefit since this sparsity property can be utilized to cancel the noise.

Lemma 1 (Moments of Stable Variable): If x is a SαS random variable, the fractional lower order moment precisely has the form

$$\mathbf{E}(|x|^p) = \begin{cases} C(p,\alpha)\gamma^{p/\alpha}, & 0 < p \leq \alpha \\ \infty, & \text{otherwise} \end{cases}$$