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Globally optimal joint design of two sets of mask coefficients in two different predefined rotated axes of time frequency plane for signal restoration applications

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A B S T R A C T

The benefits of applying two mask operations in two different rotated axes of the time frequency (TF) plane are well known especially for signal restoration applications. Compared to just applying a single mask operation in a single rotated axis of the TF plane, it has been shown that applying two mask operations in two different rotated axes of the TF plane carefully can improve the restoration performances. However, there is no systematic approach for the globally optimal joint design of these two sets of mask coefficients in two different predefined rotated axes of the TF plane. In this paper, this optimal joint design problem is formulated as a nonconvex optimization problem. Then, a modified filled function method is employed for finding the globally optimal solution of the optimization problem. Computer numerical simulation results show that the obtained restoration system outperforms existing ones.

1. Introduction

For signals having spectral contents which are invariable or changes slowly with respect to the time, filtering in the frequency domain is a powerful method for restoring signals. However, this approach is not appropriate when there is a deviation from the stationarity. This is because the modification of the components in the frequency domain has a global effect on the waveform in the time domain. For instance, for those signals whose central frequencies vary linearly with respect to the time such as the linear frequency modulated echoes occurring in radar or ultrasound applications [4], a typical bandpass filter can only attenuate the portion of the signal lying outside a horizontal strip in the TF plane. On the other hand, if a mask operation is applied in a rotated axis of the TF plane, the corresponding strip will be tilted accordingly [1–3]. Hence, this mask operation can achieve a better performance for the restoration of this signal. This is because the energy of this signal is concentrated along the strip tilted in the TF plane.

To further customize the shape of the mask, two different strips are employed by applying two different mask operations in two different rotated axes of the TF plane. It has been shown that this approach could improve the restoration performance compared to just applying a single mask operation in a single rotated axis of the TF plane [5,6,11,12]. However, the globally optimal joint design of these two sets of mask coefficients in two different predefined rotated axes of the TF plane is a nontrivial task as the objective function of the optimization problem is nonconvex. In general, it is very challenging to find the globally optimal solutions of nonconvex optimization problems [8–10]. To address this difficulty, only one set of mask coefficients is designed in each iteration while another set of mask coefficients remains unchanged. Once one set of mask coefficients is obtained, this obtained set of mask coefficients remains unchanged and another set of mask coefficients is designed. The whole design procedures are iterated until these two sets of mask coefficients converge [5,11,12]. However, there are two main drawbacks of this method. The first disadvantage of this method is that the convergence of the algorithm is in general not

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guaranteed. Second, the obtained solution is likely to be trapped in a local minimum of the optimization problem even though the algorithm converges. In order to obtain a better solution, the common approach is to repeat the entire procedures using different initial conditions and take the solution corresponding to the lowest objective functional value as the final solution. Although a better result could be obtained, this method still cannot guarantee to obtain the globally optimal solution. Also, as the objective functional value of the new trial could be higher than that of the previous trial, many computational efforts are wasted and this method is neither efficient nor effective.

This main objective and the contribution of this paper is to employ a filled function method to perform the globally optimal joint design of two sets of mask coefficients in two different predefined rotated axes of the TF plane. The working principle of using the filled function to find the globally optimal solution of an optimization problem is as follows. By finding a local minimum of the filled function at a point slightly deviated from the current local minimum of the original optimization problem, a lower objective functional value of the filled function can be obtained. Hence, the filled function could kick away from the current local minimum of the original optimization problem. Since the properties of the filled function guarantee that the current local minimum of the filled function is neither in the current basin nor in any higher basins of the original optimization problem, the current local minimum of the filled function is in a lower basin of the original optimization problem. As a result, by finding the next local minimum of the original optimization problem via searching the neighborhood around the current local minimum of the filled function, a better local minimum of the original optimization problem can be obtained. By repeating these procedures, if the original optimization problem contains a finite number of local minima, then the global minimum of the original optimization problem will be eventually reached. Since the globally optimal solution of the optimization problem is found, a better performance can be achieved.

The outline of this paper is as follows. The theoretical principles underpinning this paper are reviewed in Section 2. Section 3 presents the formulation of the design problem and the method for finding the globally optimal solution of the nonconvex optimization problem. Computer numerical simulation results are represented in Section 4. Finally, a conclusion is drawn in Section 5.

2. Review on theoretical backgrounds

A review on the discrete fractional Fourier transform (FrFT), a theoretical background on applying two mask operations in two different predefined rotated axes of the TF plane as well as a modified filled function method for finding the globally optimal solution of a nonconvex optimization problem are presented in this section. In this paper, we assume that the discrete time signals are with finite lengths. Hence, vector and matrix notations can be employed for describing the mask operations.

2.1. Discrete FrFT

The FrFT is a generalization of the ordinary Fourier transform with an order parameter a. Mathematically, the ath order FrFT operator is the ath power of the ordinary Fourier transform operator. The ath order FrFT is a linear and unitary transform which transforms the signal $x(t)$ in the time axis of the TF plane to the signal in a rotated axis of the TF plane. The transform is defined as follows [3]:

$$F^a\big[x(t)\big] = x_a(t_a) = \int B_a(t_a, t)x(t)dt, \tag{1}$$

where

$$B_a(t_a, t) = C_\phi \exp\left\{ j\pi \left(-2\frac{t_a t}{\sin\phi} + (t_a^2 + t^2)\cot\phi \right) \right\} \tag{2}$$

and $C_\phi = \frac{\exp\{-j[(\pi \operatorname{sgn}(\phi))/4-\phi/2]\}}{\sqrt{|\sin\phi|}}$. Here, $\phi = a\pi/2$ with a being a real number in the interval $0 < |a| < 2$. For $a = 1$, the FrFT is equivalent to the conventional Fourier transform.

For the FrFT, its eigen functions are the Hermite Gaussian functions $\psi_n(t)$ [3]. That is,

$$F^a\big[\psi_n(t)\big] = e^{-jan\pi/2}\psi_n(t_a), \tag{3}$$

where $\psi_n(t)$ is defined as follows:

$$\psi_n(t) = \frac{2^{1/4}}{\sqrt{2^n n!}} H_n(\sqrt{2\pi}t)e^{-\pi t^2}. \tag{4}$$

Here, $H_n(t)$ is the nth order Hermite polynomial defined as $H_n(t) = (-1)^n e^{t^2} \frac{d^n}{dt^n}(e^{-t^2})$. Since the nth order Hermite Gaussian functions can form a complete and orthonormal set of the signals in the L_2 space, they are widely used in many engineering applications.

In the discrete case, the discrete fractional Fourier transform operator is represented by matrices. The matrix representing the discrete FrFT corresponding to the FrFT with the order a is denoted as $\mathbf{F}_a$. The element in the mth row and the nth column of $\mathbf{F}_a$ for $m = 0, \ldots, N-1$ and for $n = 0, \ldots, N-1$ is defined as [7]:

$$\mathbf{F}_a[m,n] = \sum_{\substack{k=0 \\ k\neq(N-1+\operatorname{mod}(N,2))}}^{N} \mathbf{u}_k[m]e^{-j\frac{\pi}{2}ka}\mathbf{u}_k[n]. \tag{5}$$

Here, $\mathbf{u}_k[n]$ denotes the kth discrete Hermite Gaussian function. Also, N is the length of the signal. When $a = 1$, $\mathbf{F}_a$ becomes the discrete Fourier transform (DFT) matrix. The discrete FrFT has the similar properties of the DFT such as:

i. linearity, that is, $\mathbf{F}_a(\mathbf{x}+\mathbf{y}) = \mathbf{F}_a(\mathbf{x}) + \mathbf{F}_a(\mathbf{y})$;
ii. unitarity, that is, $\mathbf{F}_a^H = \mathbf{F}_a^{-1}$, where $\mathbf{F}_a^H$ is the conjugate transpose of $\mathbf{F}_a$;
iii. index additivity, that is, $\mathbf{F}_a\mathbf{F}_b = \mathbf{F}_{a+b}$.

2.2. Applying two mask operations in two different predefined rotated axes of the TF plane

In TF representations, the TF domain is a set consisting of the ordered pairs of time and frequency. It is graphically represented by a plane with the x axis being the time axis and the y axis being the frequency axis. The Fourier transform is to represent signals in the y axis of the TF plane. Here, the y axis is the axis obtained by rotating the time axis of the TF plane by 90°. On the other hand, a FrFT [1] is to represent signals in a rotated axis of the TF plane, in which the axis is obtained by rotating the time axis of the TF plane by a certain angle. By applying a single mask operation in a single rotated axis in the TF plane, useful signal components can be extracted out. When the rotational angle is 90°, the FrFT becomes the conventional Fourier transform. Since filtering is to apply a mask operation in the y axis of the TF plane, this mask operation becomes the conventional filtering. From here, we can see that the FrFT is a generalization of the conventional Fourier transform and applying a mask operation in the rotated axis of the TF plane is a generalization of filtering in the frequency domain. Let $\mathbf{y}$ and $\mathbf{z}$ be the signals represented in the time domain and in the FrFT domain with the order a, respectively. That is, $\mathbf{z} = \mathbf{F}_a\mathbf{y}$. Then, a mask operation is applied to $\mathbf{z}$ in the FrFT domain. Let the vector of mask coefficients be $\mathbf{g}$ and the signal after applying the mask operation