Digital Signal Processing 40 (2015) 181–197

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

Localization of multiple disjoint sources with prior knowledge on source locations in the presence of sensor location errors

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ARTICLE INFO

Article history:
Available online 18 February 2015

Keywords:
Multiple disjoint source localization
Gaussian-distributed prior source locations
Time difference of arrival (TDOA)
Frequency difference of arrival (FDOA)
Cramér–Rao lower bound (CRLB)

ABSTRACT

Sensor location errors are known to be able to degrade the source localization accuracy significantly. This paper considers the problem of localizing multiple disjoint sources where prior knowledge on the source locations is available to mitigate the effect of sensor location uncertainty. The error in the priorly known source location is assumed to follow a zero-mean Gaussian distribution. When a source location is completely unknown, the covariance matrix of its prior location would go to infinity. The localization of multiple disjoint sources is achieved through exploring the time difference of arrival (TDOA) and the frequency difference of arrival (FDOA) measurements. In this work, we derive the Cramér–Rao lower bound (CRLB) of the source location estimates. The CRLB is shown analytically to be able to unify several CRLBs introduced in literature. We next compare the localization performance when multiple source locations are determined jointly and individually. In the presence of sensor location errors, the superiority of joint localization of multiple sources in terms of greatly improved localization accuracy is established. Two methods for localizing multiple disjoint sources are proposed, one for the case where only some sources have prior location information and the other for the scenario where all sources have prior location information. Both algorithms can reach the CRLB accuracy when sensor location errors are small. Simulations corroborate the theoretical developments.

1. Introduction

Determining the source positions from source signal measurements obtained by an array of sensors at a time instance is a classic problem. It has found many applications in radar, sonar, search and rescue. It is of practical importance especially for the scenario where only a single set of source signal measurements is available for the localization task, due to e.g., the small duty cycle of the sources.

Source localization is a non-trivial problem, mainly because the source positions and the signal measurements are generally nonlinearly related. Over the past few decades, a number of localization algorithms have become available in literature. To name a few, they include the iterative Taylor-series method [1,2], the two-step least-squares (TSLS) method [3–5], the linear-correction least-squares (LCLS) method [6] and other closed-form techniques (see [7–12] and references therein). Recently, the use of the Laplacian mixture model in speaker localization was investigated in [13]. Dehkordi et al. [14] explored the spatial sparsity of the source localization problem and developed a compressive sensing (CS) based localization technique. Wei et al. [15] introduced a multidimensional scaling (MDS)-based approach for source localization. The MDS method was shown to be more robust to large noise levels but it cannot attain the Cramér–Rao lower bound (CRLB) accuracy when the noise level is small. The localization algorithms mentioned above all assume that the sensor locations known for the source localization task are accurate, which may not hold in practice. In [16], Ho et al. proposed a localization method that takes the sensor location errors into account to improve performance. Although the new solution can reach the CRLB approximately, the obtained localization accuracy is quite worse than that of the case where the sensor locations are known precisely.

In order to mitigate the effect of sensor location errors on the accuracy of locating a source from time difference of arrival (TDOA) measurements, Ho and Yang [17] considered using a single calibration emitter. They showed via CRLB analysis that the use of a

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http://dx.doi.org/10.1016/j.dsp.2015.02.003
1051-2004/

calibration emitter at accurately known position can significantly improve the source localization performance. An approximately efficient closed-form method that can explore the TDOAs from both the unknown source and the calibration emitter was also developed. Yang and Ho [18] later extended their work in [17] to a more realistic situation where there exist more than one calibration emitters and their positions are subject to errors as well. Recently, the study in [18] was further generalized in [19], where the use of calibration sensors for reducing the impact of sensor location errors was investigated.

When the calibration emitters in the source localization scenario considered in [18] are at completely unknown positions, the calibration emitter positions need to be determined together with the source position. This leads to the problem of locating multiple disjoint sources. The disjointness could come from time, frequency or both [12] so that the signal measurements from every source can be obtained separately. This eliminates the data association procedure [20–22] usually required in non-disjoint source localization. In [23], an approximately efficient technique has been developed for locating multiple disjoint sources using TDOAs in the presence of sensor position errors. When the received source signal strength is also available, the gain ratio of arrival (GROA) can be explored together with TDOAs to accomplish multiple disjoint source localization [24,25]. In the case where the source and the sensors are moving, [26] developed a closed-form algorithm for estimating the positions and the velocities of multiple disjoint sources from TDOA and frequency difference of arrival (FDOA) measurements obtained by sensors whose positions and velocities are known imprecisely. Iterative methods are also available for the multiple disjoint localization task (see [27] for an example). They utilized numerical techniques such as the Taylor-series method to solve the maximum likelihood (ML) estimation problem that is nonlinear with respect to the source locations. The iterative methods may suffer from local convergence or even divergence if their initial solution guesses are far from the true source locations.

In this paper, we consider the problem of locating multiple disjoint sources from the source TDOA and FDOA measurements when the sensor locations are subject to random errors. Different from [26], this work assumes that a noisy version of the source locations is available and they are referred to as prior source locations. The deviation of the prior source locations from their true values is assumed to be Gaussian distributed. The above localization setup includes three application scenarios as special cases. First, when the covariance matrices of the prior source locations are infinitely large, i.e., all source locations are completely unknown, the considered localization problem reduces to the one studied in [26]. Second, when only some sources are at completely unknown locations (i.e., their prior locations have infinitely large covariance matrices while others have finite covariance matrices), the problem becomes multiple disjoint source localization with calibration emitters at imprecisely known locations. This can be considered as a generalization of the problem studied in [18]. More specifically, we shall investigate the localization of multiple disjoint sources using TDOAs and FDOAs, compared to [18] where the TDOA-positioning of a single source was considered. This problem has not been examined carefully in literature except for [28] where some preliminary results were presented. In the third case, all prior source locations have finite covariance matrices. The prior source location information can be explored together with the TDOA and FDOA measurements to provide improved source localization accuracy.

The main contributions of this paper are:

(1) The CRLB of localizing multiple disjoint sources is derived for the scenario where sensor location errors and prior source locations are present. We demonstrate, via manipulating the covariance matrices of the prior source locations, that several localization CRLBs in literature can be deduced from the newly obtained CRLB. This indicates that the localization problem investigated in this work is more general and the difference among the previously studied localization scenarios mainly lies in the availability of the prior source location information.
(2) On the basis of the newly derived localization CRLB, we conduct a theoretical study that compares the localization accuracy when multiple sources are localized jointly and individually. It is found that when sensor location errors are present, joint source localization outperforms locating the sources separately in terms of significantly improved localization accuracy. On the other hand, in the absence of sensor location errors, locating multiple disjoint sources together and individually would yield the same performance.
(3) Two algorithms for localizing multiple disjoint sources are proposed. The first algorithm addresses the scenario where only some sources have prior location information. It is a generalization of the method developed in [18]. It uses the sources with prior location information as calibration emitters and localizes other sources at completely unknown locations. The second algorithm tackles the situation where all sources have prior location information. Both proposed techniques are shown to be able to achieve the CRLB under mild conditions.

The rest of the paper is organized as follows. Section 2 describes the localization scenario. Section 3 derives the source localization CRLB and performs the CRLB analysis. Section 4 presents the two methods for jointly estimating the locations of multiple disjoint sources. Section 5 contains the simulation results. Section 6 concludes the paper. The following notations are used throughout the paper. Bold face upper case letter denotes matrix and bold face lower case letter represents vector. Besides, for a noisy quantity $\{\cdot\}$, we shall denote its noise-free version (i.e., the true value) as $\{\cdot\}^o$ and the random error in $\{\cdot\}$ as $\Delta\{\cdot\}$ such that $\{\cdot\} = \{\cdot\}^o + \Delta\{\cdot\}$.

2. Localization scenario

This paper considers the localization scenario depicted in Fig. 1. There are N moving sources whose positions and velocities are to be determined. Let $\mathbf{u}_i^o = [u_{x,i}^o, u_{y,i}^o, u_{z,i}^o]^T$ and $\dot{\mathbf{u}}_i^o = [\dot{u}_{x,i}^o, \dot{u}_{y,i}^o, \dot{u}_{z,i}^o]^T$ be the true position and the true velocity of source i, $i = 1, 2, \ldots N$. We define $\boldsymbol{\theta}_i^o = [\mathbf{u}_i^{oT}, \dot{\mathbf{u}}_i^{oT}]^T$ as the true location vector of source i. In this work, it is assumed that a noisy version of $\boldsymbol{\theta}_i^o$, denoted by $\boldsymbol{\theta}_i$ and referred to as the prior source location, is available. The deviation of $\boldsymbol{\theta}_i$ from $\boldsymbol{\theta}_i^o$, $\Delta\boldsymbol{\theta}_i = \boldsymbol{\theta}_i - \boldsymbol{\theta}_i^o$, is modeled as a zero-mean Gaussian random vector with covariance matrix $\mathbf{Q}_{\boldsymbol{\theta}_i}$. Note that when $\mathbf{Q}_{\boldsymbol{\theta}_i}$ goes to infinity, the above setting reduces to the scenario where the true location of source i is completely not known. We define the composite source location vector as $\boldsymbol{\theta}^o = [\boldsymbol{\theta}_1^{oT}, \boldsymbol{\theta}_2^{oT}, \ldots, \boldsymbol{\theta}_N^{oT}]^T$. Mathematically, we have $\boldsymbol{\theta} = \boldsymbol{\theta}^o + \Delta\boldsymbol{\theta}$, where $\boldsymbol{\theta} = [\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_2^T, \ldots, \boldsymbol{\theta}_N^T]^T$ and $\Delta\boldsymbol{\theta} = [\Delta\boldsymbol{\theta}_1^T, \Delta\boldsymbol{\theta}_2^T, \ldots, \Delta\boldsymbol{\theta}_N^T]^T$. It is assumed that $\Delta\boldsymbol{\theta}_i$ are independent to one another such that the covariance matrix of $\Delta\boldsymbol{\theta}$ is $\mathbf{Q}_{\boldsymbol{\theta}} = diag\{\mathbf{Q}_{\boldsymbol{\theta}_1}, \mathbf{Q}_{\boldsymbol{\theta}_2}, \ldots, \mathbf{Q}_{\boldsymbol{\theta}_N}\}$.

There are M mobile sensors. Their true positions and velocities are collected in $\mathbf{s}^o = [\mathbf{s}_1^{oT}, \mathbf{s}_2^{oT}, \ldots, \mathbf{s}_M^{oT}]^T$ and $\dot{\mathbf{s}}^o = [\dot{\mathbf{s}}_1^{oT}, \dot{\mathbf{s}}_2^{oT}, \ldots, \dot{\mathbf{s}}_M^{oT}]^T$, where $\mathbf{s}_j^o = [s_{x,j}^o, s_{y,j}^o, s_{z,j}^o]^T$ and $\dot{\mathbf{s}}_j^o = [\dot{s}_{x,j}^o, \dot{s}_{y,j}^o, \dot{s}_{z,j}^o]^T$ are the true position and the true velocity of sensor j, $j = 1, 2, \ldots, M$. Both $\mathbf{s}^o$ and $\dot{\mathbf{s}}^o$ are not known and only their noisy values $\mathbf{s} = [\mathbf{s}_1^T, \mathbf{s}_2^T, \ldots, \mathbf{s}_M^T]^T$ and $\dot{\mathbf{s}} = [\dot{\mathbf{s}}_1^T, \dot{\mathbf{s}}_2^T, \ldots, \dot{\mathbf{s}}_M^T]^T$ are available. The sensor position error and the sensor velocity error are denoted by $\Delta\mathbf{s} = [\Delta\mathbf{s}_1^T, \Delta\mathbf{s}_2^T, \ldots, \Delta\mathbf{s}_M^T]^T$ and $\Delta\dot{\mathbf{s}} = [\Delta\dot{\mathbf{s}}_1^T, \Delta\dot{\mathbf{s}}_2^T, \ldots, \Delta\dot{\mathbf{s}}_M^T]^T$ such that $\mathbf{s} = \mathbf{s}^o + \Delta\mathbf{s}$ and $\dot{\mathbf{s}} = \dot{\mathbf{s}}^o + \Delta\dot{\mathbf{s}}$. Define the sensor location vector as