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Detrended fluctuation thresholding for empirical mode decomposition based denoising

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ABSTRACT

Signal decompositions such as wavelet and Gabor transforms have successfully been applied in denoising problems. Empirical mode decomposition (EMD) is a recently proposed method to analyze non-linear and non-stationary time series and may be used for noise elimination. Similar to other decomposition based denoising approaches, EMD based denoising requires a reliable threshold to determine which oscillations called intrinsic mode functions (IMFs) are noise components or noise free signal components. Here, we propose a metric based on detrended fluctuation analysis (DFA) to define a robust threshold. The scaling exponent of DFA is an indicator of statistical self-affinity. In our study, it is used to determine a threshold region to eliminate the noisy IMFs. The proposed DFA threshold and denoising by DFA–EMD are tested on different synthetic and real signals at various signal to noise ratios (SNR). The results are promising especially at 0 dB when signal is corrupted by white Gaussian noise (WGN). The proposed method outperforms soft and hard wavelet threshold method.

1. Introduction

The empirical mode decomposition (EMD) is an alternative method to analyze non-linear and non-stationary signals [1]. EMD breaks signal down into a finite number of amplitude and frequency modulated (AM/FM) zero-mean oscillations called intrinsic mode functions (IMFs). In contrast to wavelet decomposition, IMFs are expressed as the signal dependent semi-orthogonal basis functions via an iterative algorithm called sifting. However, they have fluctant frequency spectrum caused by mode-mixing effect. There are several attempts to reduce fluctuation or express an IMF with a single component [2]. On the other hand, it is another challenging study to explain the meaning of each IMF or determine which IMF refers to noisy oscillations, which is the generalized task of the EMD based denoising. While noisy IMFs may be determined manually observing the periodicity of the oscillations in the required range [3], some automated methods have been studied. Wu and Huang [4] deployed a hypothesis test method to find out the relevant information level of the IMFs, which is reported to perform poorly for low frequency oscillations. Information theoretical based approaches such as mutual information and relative entropy are applied to find noisy oscillations [5,6]. In addition to these time-domain characteristics of IMFs, frequency domain characteristics of the EMD is investigated and modeled as a dyadic filter bank resulting from the decomposition of white Gaussian noise (WGN) [7] or fractional Gaussian noise (fGn) [8,9]. From this point of view, if the energy distribution of IMFs for noise-only signal is known, the discrepancy between energy of noisy-signal IMFs and noise-only IMFs indicates the presence of the relevant informative oscillations.

Our suggestion is to determine noisy IMF resulting from the decomposition of noisy-signal using a reliable metric. Detrended fluctuation analysis (DFA) [10] is a successful method to measure long-range dependency for non-stationary time series [11,12]. The special cases $\alpha = 0.5$, $\alpha = 1$ and $\alpha = 1.5$ correspond to completely uncorrelated white noise, pink noise and Brownian noise. When $0 < \alpha < 0.5$, the signal is called “anti-correlated”, in which large fluctuations are likely to be followed by small ones. While it increases from 0.5 to 1, temporal correlations are persistent. If $\alpha > 1$, the correlations do not exhibit power-law behavior [13]. The slope, α can also be considered as an indicator of roughness [14]: the larger the value, the smoother time series or slower fluctuations. From this point of view, DFA can be used as a robust metric to identify noisy IMFs. The proposed method is to determine noisy IMFs resulting from the decomposition of noisy-signal using a reliable metric which is independent of a comparison or referencing with the signal. The IMFs are tested by DFA to measure their statistical properties, and the DFA based threshold is applied to exclude

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IMFs which contain mostly noise. The suggested method is tested on synthetic and real EEG signals to show its denoising capability comparing to wavelet threshold methods.

The remainder of the paper is organized as follows: Section 2 provides a short description of EMD. Signal denoising and thresholding are described in Section 3. Section 4 summarizes the DFA and explores its thresholding capability. The suggested DFA–EMD based denoising is presented in Section 5. Consequently, in Section 6 simulation results of the DFA–EMD method are examined, and the conclusions are drawn in Section 7.

2. EMD: a brief description

The EMD has been introduced by Huang et al. [1] as a tool of data driven and adaptive multi-component signal decomposition method into intrinsic mode functions (IMFs) so that sum of them is equal to the original signal $x(n)$. IMFs are required to satisfy two criteria [2]: First, the number of the extrema and the number of zero crossings must be equal or must differ by one at most. Second, the mean of the upper and lower envelopes determined by the local maxima and minima should be zero. The most important iterative process of an EMD algorithm is to extract IMFs called *Sifting*, which is composed of the following steps [15]:

(i) Find local maxima, M_i, $i = 1, 2, \ldots,$ and minima m_k, $k = 1, 2, \ldots,$ in $x(n)$.
(ii) Compute the interpolating signals $M(n) = f_M(M_i, n)$, and $m(n) = f_m(m_k, n)$ using cubic spline, which are the upper and lower envelopes of the signal.
(iii) Compute mean of the envelopes, $e(n) = [M(n) + m(n)]/2$.
(iv) If $e(n)$ satisfies the IMF requirements, save it as an IMF, and remove $e(n)$ from the signal; $x(n) = x(n) - e(n)$.
(v) Return to step (i) and stop when $x(n)$ remains nearly unchanged.
(vi) After obtaining an IMF, $\varphi_i(n)$, remove IMF from the signal $x(n) = x(n) - \varphi_i(n)$ and return to (i) if $x(n)$ is not constant or trend, $r(n)$.

Consequently, the original signal can be reconstructed by the sum of IMFs described as follows;

$$x(n) = \sum_{i=1}^{L} \varphi_i(n) + r(n) \tag{1}$$

where L is the number of extracted IMFs. There are several EMD algorithms in which different sifting methods are deployed to enhance its capabilities. The first implementation of the EMD algorithm deploys standard deviation (SD) based approach to guarantee that IMFs retain sufficient physical sense of amplitude and frequency modulation is defined as

$$SD = \sum_{n=0}^{N} \left[\frac{|e_{(k-1)}(n) - e_{(k)}(n)|^2}{e_{(k-1)}^2(n)} \right] \tag{2}$$

where k denotes the iteration number in the sifting algorithm. It was reported that SD should be chosen between 0.2 and 0.3 for meaningful results [1]. A recent EMD algorithm [15] used in this study suggests an alternative stopping criteria by predefined resolutions;

$$qResol = 10 \log\left(\frac{\sigma_{x(n)}^2}{\sigma_{e(n)}^2} \right) \tag{3}$$

$$qResid = 10 \log\left(\frac{\sigma_{x(n)}^2}{\sigma_{r(n)}^2} \right) \tag{4}$$

where *qResol* and *qResid* are the ratios of signal to IMF and signal to residue energy, respectively. Thus, an oscillation may be assigned as an IMF, or residue, and it may increase the iteration number. There are some studies to enhance the sifting algorithm to eliminate the drawbacks caused by interpolation such as "end effect", and "mode-mixing" [15–18]. Besides, the EMD is successfully applied to the problems such as denoising [8,9] instantaneous frequency [19], autoregressive parameter estimation [20], classification [21], and audio coding [22].

3. Signal denoising

A common description of a denoising problem can be described as follows: A sampled noisy signal $x(n)$ is obtained by

$$x(n) = \bar{x}(n) + \sigma\eta(n), \quad t = 1, 2, \ldots, N \tag{5}$$

where $\bar{x}(n)$ is the noise free signal and $\eta(n)$ is Gaussian distributed $N(0, 1)$ independent random variable with known or unknown noise variance σ. The goal is to recover an estimated version $\tilde{x}(n)$ of $\bar{x}(n)$ with small error. The performance criteria may be mean squared error (MSE), $MSE = \frac{1}{N}\|\tilde{x}(n) - \bar{x}(n)\|_2^2$ or signal to noise ratio (SNR), $SNR = 10\log(\frac{\sigma_{\bar{x}}^2}{\sigma_\eta^2})$.

The main principle of the wavelet denoising is to apply threshold to the resultant coefficients in the orthogonal basis. Coefficients with higher amplitudes than threshold are assumed as the $\bar{x}(n)$ related components. Because, total energy of the noiseless signal is represented by a few coefficients in the wavelet domain, and the others are assumed as noise components. There are two major threshold methods called hard and soft threshold defined as

$$\rho_T(c) = \begin{cases} c, & |c| > \theta \\ 0, & |c| \le \theta \end{cases} \tag{6}$$

$$\rho_T(c) = \begin{cases} sgn(c)\,(|c| - \theta), & |c| > \theta \\ 0 & |c| \le \theta \end{cases} \tag{7}$$

respectively, where $\rho_T(c)$ denotes the thresholded wavelet coefficients which are applied to inverse wavelet transform to recover noise free $\bar{x}(n)$, and the others lower than threshold are set to zero. The hard thresholding is the simplest way and sets all values to zero if below then the threshold. The soft threshold method also known as wavelet shrinkage, shrinks the coefficients with higher amplitude towards zero [23,24]. In addition to these threshold methods, the other criterion affecting the denoising capability is the selection of the level of the threshold. Briefly, the universal threshold $\theta = \hat{\sigma}\sqrt{2\ln N}$ is the most popular candidate. The standard deviation of the noise is estimated applying a robust estimator defined as [25].

$$\hat{\sigma} = \frac{median(|c_i|: i = 1, \ldots, N)}{0.6745} \tag{8}$$

Such a threshold computed based on noisy signal coefficients guarantees that components with lower amplitudes belong to the noise. Therefore, sufficient denoising performance is obtained independently from noise characteristics [26]. On the other hand, aforementioned methods such as threshold and its level depend on the estimation using the properties of the signal and noise. This is the reason of several attempts to fit more appropriate estimators. Moreover, a-priori basis and decomposition level selection make it a trial and error based method to obtain the best denoising performance. Decomposition of a noisy signal by EMD reveals both noise and noise free IMFs. The goal is to determine a reliable metric to discriminate the noise components and exclude them when reconstructing the signal. Fig. 1 summarizes the method in the EMD based denoising. A piecewise-regular signal with 10 dB SNR