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On the regular precession of an asymmetric rigid body acted upon by uniform gravity and magnetic fields

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ABSTRACT

In 1947 Grioli discovered that an asymmetric heavy rigid body moving about a fixed point can perform a regular precession, which is the rotation of the body about an axis fixed in it, while that axis precesses with the same uniform angular velocity about a non-vertical axis fixed in space.

In the present note, we show that a magnetized asymmetric rigid body moving about a fixed point while acted upon by uniform gravity and magnetic fields can perform a regular precession about a horizontal axis fixed in space orthogonal to the magnetic field. This motion does not contain Grioli's as a special case since the gravity and magnetic effects are coupled and can vanish only simultaneously.

1. Introduction

1.1. Historical

The subject of rigid body dynamics is two and half centuries old. It dates back to Euler, who introduced the basic notions and studied the motion of the torque-free body [1] (1758). Lagrange studied the case of an axi-symmetric body (top) in the uniform gravity field [2] (1788). It was found later that both cases of Euler and Lagrange have their general solutions as elliptic functions of the time variable t. The equations of motion are usually written in the form known as the Euler–Poisson equations and they admit three general integrals of motion: the total energy, the integral of areas and the geometric integral. The integrability of those equations requires the knowledge of a complementary (fourth) integral of motion, independent of those three (see e.g. [4]).

A whole century elapsed after Lagrange's work before Kowalevski found a third integrable case, now known after her name [3] (1889). She isolated this case by an interesting property: only in those three cases the general solution of the equations of motion of the heavy rigid body about a fixed point can be expressed for all initial conditions in terms of functions that have no singularities other than poles in the complex plane of the time variable t. She also found the complementary integral, which turned out to be, for the first time in

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dynamics, a polynomial of degree 4 in the angular velocity components and constructed the explicit solution in ultra-elliptic functions of time.

Goryachev and Chaplygin constructed the fourth integral for one more case. This case is conditionally integrable, i.e. integrable only on the zero level of the areas integral. This dynamical condition means that the motion is integrable only when the angular momentum of the body lies permanently in a horizontal plane. The conditional complementary integral for this case is a cubic polynomial in the angular velocities and explicit solution is expressed also in ultra-elliptic functions of time (e.g. [4]).

Kowalevski's results created a great interest all along the next century in exploring the deep relation between the branching of the solution of equations of motion in the complex $t-$ plane and algebraic integrability, i.e. existence of the fourth integral as a polynomial or algebraic function of the phase space. This research began with the works of Liouville, Husson and Burgatti [6–8] and was culminated with results of Kozlov and Ziglin [9–11]. In [10], it is shown that a meromorphic general integral of the equations of motion exists only in the cases of Euler, Lagrange and Kowalevski; and a conditional one only in the case of Goryachev and Chaplygin. For a review of those and related results, see e.g. [5].

After Kowalevski, the interest in the problem has shifted to the search for particular solutions of the equations of motion. Those are solutions under any conditions on the initial state of motion as well as on the distribution of mass in the body. The search for particular solutions produced 11 solutions, which, with the well-known motion of the body as a composite pendulum complete the list of 12 cases shown in the following Table 1:

For a detailed account of those cases see [25] or [26]. The last of them was found in 1970. Some of those solutions were generalized later through the addition of a gyrostatic moment and new solutions for a gyrostat were found by several authors: N.E.Zhoukovski, H.M.Yehia, L.N.Sretensky, D.N.Goryachev, A.I.Dokshevich, L.M.Kovaleva, G.V.Mozalevskaya, P.V.Kharlamov, E.I.Kharlamova. For the details see [25] and references therein.

Hess' case had a wide generalization including a gyrostatic moment and other potential and gyroscopic forces [27]. Grioli's case was also generalized to include an additional parameter, which transforms it into a solvable case of the dynamics of a rigid body by inertia in a fluid [28].

A direct, simple but very important generalization of the problem described above is that of motion of a rigid body under the action of a combination of two uniform fields. This problem is characterized by two vectors, constant in space which represent gravity and magnetic fields and two vectors, constant in the body, describing the centre of mass and the magnetic moment. The potential of this problem is a linear function in all the direction cosines of the two fields with respect to the body frame.

In spite of its practical importance, the problem of motion of rigid body under the action of more than one uniform fields has escaped attention for a long time. Despite the richness in its structure, integrable cases of this problem are still rare. Till now, none of the above results concerning properties of solutions in the complex plane of time or the existence of algebraic integrals could be generalized to cover this problem. The research in this problem was not carried out on a systematic basis, and only scattered results exist.

Although the integrals of motion were found so early as in 1893 in a much more complicated problem of motion of a rigid body influenced by the approximate Newtonian field of three attraction centres non-coplanar with the fixed point [29], the problem of motion of a rigid body influenced by constant gravity and magnetic fields was considered almost a century later, namely in 1984, by Bogoyavlensky [32]. He established that for Kowalevski's configuration $A = B = 2C$, this problem is Liouville integrable on a submanifold characterized by two invariant relations of the second degree. In our notation, this is equivalent to construction of a particular solution of the equations of motion. Shortly later, in our work [33] of 1986, we constructed a fourth-degree integral, which generalizes the famous integral of Kowalevski for the classical problem of one field by adding the second field and, simultaneously, attaching a gyrostatic moment. One more integral was still lacking to establish integrability in the new problem, since this problem admits no cyclic integral in general. In the same work [33], we isolated another integrable version of the problem with a cyclic integral corresponding to the sum (or difference) of the two angles of precession and proper rotation. This version does not stem out of Kowalevski's case, in the sense that it does not include that case of one field as a particular case,

Table 1 – Known particular solvable cases of the classical problem (in chronological order).

Case	1	2	3	4
Au.	Pendulum motion	Hess	Staude	Bobylev-Steklov
Year	–	1890	1894	1896
Ref.	–	[12]	[13]	[14,15]
Case	5	6	7	8
Au.	Goryachev	Steklov	Chaplygin	Kowalewski
Year	1899	1899	1904	1908
Ref.	[18]	[16]	[17]	[19]
Case	9	10	11	12
Au.	Grioli	Dokshevich	Konosevich-Pozdnyakovich	Dokshevich
Year	1947	1965	1968	1970
Ref.	[20]	[21]	[22]	[23]