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Exact travelling wave solutions of the coupled nonlinear evolution equation via the Maccari system using novel (G′/G)-expansion method

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ABSTRACT

In this article, the novel (G′/G)-expansion method is used to construct exact travelling wave solutions of the coupled nonlinear evolution equation. This technique is uncomplicated and simple to use, and gives more new general solutions than the other existing methods. Also, it is shown that the novel (G′/G)-expansion method, with the help of symbolic computation, provides a straightforward and vital mathematical tool for solving nonlinear evolution equations. For illustrating its effectiveness, we apply the novel (G′/G)-expansion method for finding the exact solutions of the (2 + 1)-dimensional coupled integrable nonlinear Maccari system.

1. Introduction

The investigation of exact travelling wave solutions to nonlinear evolution equation plays an important role in the study of nonlinear physical phenomena for various fields of science and engineering, especially in mathematical physics, plasma physics, fluid dynamics, quantum field theory, biophysics, chemical kinematics, geochemistry, propagation of shallow water waves, high-energy physics and so on. The analytical solutions of such equations are of fundamental

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importance since a lot of mathematical-physical models are described by nonlinear evolution equations (NLEEs). Many powerful and direct methods have been developed to find explicit solutions to the NLEEs, such as, wave of translation [1], the inverse scattering transform [2], the Hirota's bilinear method [3], the Darboux transformation method [4], the Backlund transformation method [5], the tanh method [6], the tanh-sech method [7], the symmetry method [8], the Painleve expansion method [9], the Exp-function method [10–14], the Adomian decomposition method [15], the homogeneous balance [16] and so on to construct exact solution of NLEEs. Lately, Wang et al. [17] introduced an expansion technique called the (G'/G)-expansion method, and they verified that it is a simple technique look for analytic solutions of NLEEs. In order to show the efficiency of the (G'/G)-expansion method and to extend the range of its applicability, further research has been carried out by several researchers, such as, Zhang et al. [18] proposed a generalization of the (G'/G)-expansion method for solving the evolution equations with variable coefficients. Zhang et al. [19] also presented an improved (G'/G)-expansion method to seek general traveling wave solutions. Zayed [20] obtainable a new approach of the (G'/G)-expansion method where $G(\xi)$ satisfies the Jacobi elliptical equation $[G'(\xi)]^2 = e_2G^4(\xi) + e_1G^2(\xi) + e_0$. Zayed [21] again proposed an alternative approach of this method in which $G(\xi)$ satisfies the Riccati equation $G'(\xi)=A+BG^2(\xi)$, where A and B are arbitrary constants. Akbar et al. [22] proposed a generalized and improved (G'/G)-expansion method which give more new solutions than the improved (G'/G)-expansion method [19]. Recently, Alam et al. [23] further improved the (G'/G)-expansion method known as novel (G'/G)-expansion method. They have solved only single NLEEs using this method.

The nonlinear Maccari system is an important mathematical model in physics. Currently, Lee et al. [24], Hafez et al. [25], and Manafian et al. [26,27] have solved the Maccari system using the Kudryashov method, the $\exp(-(\Phi(\xi)))$-expansion method and the Exp-function method respectively. Therefore, the aim of this article is to investigate new exact travelling wave solutions to the Maccari system by use of the novel (G'/G)-expansion method, which is more effective than others methods.

2. Description of the novel (G'/G)-expansion method

Let us consider the nonlinear evolution equation

$$P(u,\ u_t,\ u_x,\ u_y,\ u_{xx},\ u_{yy},\ u_{tt},\ u_{tx},\ldots), \tag{1}$$

where P is a polynomial in $u(x,y,t)$ and its partial derivatives wherein the highest order partial derivatives and the nonlinear terms are concerned. The most important steps of the method are as follows:

Step 1: Combine the real variables x,y and t by a complex variable ξ, we suppose that

$$u(x,y,t) = u(\xi), \quad \xi = x + y \pm ct, \tag{2}$$

where c denotes the speed of the traveling wave. By use of Eq. (2), Eq. (1) is converted into an ODE for $u=u(\xi)$:

$$Q(u, u', u'', u''', \cdots) = 0, \tag{3}$$

where, Q is a function of $u(\xi)$ and its derivatives wherein prime stands for derivative with respect to ξ.

Step 2: Assume the solution of Eq. (3) can be expressed in powers $\psi(\xi)$:

$$u(\xi) = \sum_{j=-N}^{N} \alpha_j(\psi(\xi))^j \tag{4}$$

where

$$\psi(\xi) = (d + \Phi(\xi)) \tag{5}$$

and $\Phi(\xi) = \frac{G'(\xi)}{G(\xi)}$.

Here α_{-N} or α_N may be zero, but both of them could not be zero simultaneously. α_j ($j=0,\pm1,\pm2,\cdots,\pm N$) and d are constants to be determined later, and $G=G(\xi)$ satisfies the second order nonlinear ODE:

$$GG'' = \lambda GG' + \mu G^2 + \nu(G')^2 \tag{6}$$

where prime denotes the derivative with respect ξ; λ, μ, and ν are real parameters.

The Cole-Hopf transformation $\Phi(\xi) = \ln(G(\xi))_\xi = \frac{G'(\xi)}{G(\xi)}$ reduces the Eq. (6) into Riccati equation:

$$\Phi'(\xi) = \mu + \lambda\Phi(\xi) + (\nu - 1)\Phi^2(\xi) \tag{7}$$

Eq. (7) has individual twenty five solutions (see Zhu [28] for details).

Step 3: The value of the positive integer N can be determined by balancing the highest order linear terms with the nonlinear terms of the highest order come out in Eq. (3). If the degree of $u(\xi)$ is $D[u(\xi)]=n$, then the degree of the other expressions will be as follows:

$$D\left[\frac{d^pu(\xi)}{d\xi^p}\right] = n + p, \quad D\left[u^p\left(\frac{d^qu(\xi)}{d\xi^q}\right)^s\right] = np + s(n + q).$$

Step 4: Substitute Eq. (4) including Eqs. (5) and (6) into Eq. (3), we obtain polynomials in $\left(d + \frac{G'(\xi)}{G(\xi)}\right)^j$ and $\left(d + \frac{G'(\xi)}{G(\xi)}\right)^{-j}$, ($j=0,1,2,\cdots,N$). Collect each coefficient of the resulted polynomials to zero, yields an over-determined set of algebraic equations for α_j ($j=0,\pm1,\pm2,\cdots,\pm N$), d and V.

Step 5: Suppose the value of the constants can be obtained by solving the algebraic equations obtained in Step 4. Substituting the values of the constants together with the solutions of Eq. (6), we will obtain new and comprehensive exact traveling wave solutions of the nonlinear evolution Eq. (1).

Discussion 1: It is noteworthy to examine that if we replace λ by $-\lambda$ and μ by $-\mu$ and put $\nu=0$ in Eq. (6), then the novel (G'/G)-expansion method coincide with Akbar et al.'s [19] generalized