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An adaptive method with weight matrix as a function of the state to design the rotatory flexible system control law

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ABSTRACT

One of the most well-known techniques of optimal control is the theory of Linear Quadratic Regulator (LQR). This method was originally applied only to linear systems but has been generalized for non-linear systems: the State Dependent Riccati Equation (SDRE) technique. One of the advantages of SDRE is that the weight matrix selection is the same as in LQR. The difference is that weights are not necessarily constant: they can be state dependent. Then, it gives an additional flexibility to design the control law. Many are applications of SDRE for simulation or real time control but generally SDRE weights are chosen constant so no advantage of this flexibility is taken. This work serves to show through simulation that state dependent weights matrix can improve SDRE control performance. The system is a non-linear flexible rotatory beam. In a brief first part SDRE theory will be explained and the non-linear model detailed. Then, influence of SDRE weight matrix associated with the state Q will be analyzed to get some insight in order to assume a state dependent law. Finally, these laws are tested and compared to constant weight matrix Q. Based on simulation results; one concludes showing the benefits of using an adaptive weight Q rather than a constant one.

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1. Introduction

The main interest of the SDRE method [1] is that it is a systematic approach that can deal with non-linear plant. A good state of the art about SDRE theory can be found in [2]. The idea of SDRE is to linearize the plant around the instantaneous point of operation, producing a constant state-space model and then calculating the controller as in the LQR control technique [3]. The process is repeated at each sampling periods producing and controlling several state dependent linear models out of a non-linear one. In other words, a SDRE controller is an adaptive LQR. Feasibility in real time could be a problem as the computation time for calculating the controller (solving the Algebraic Riccati Equation ARE) has to be inferior to the sampling time of the system. Therefore, several simulations have proven the computationally feasibility for real time implementation as in control of missiles [4] and helicopter [5]. A different approach, also based on an optimization of weight matrix, was applied by [6] and [7] to design a control system of flexible satellites. As feasibility can no more be proved, therefore, this study will focus on simulation and will show benefit of a non-linear weighting selection.

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2. SDRE methodology

One of the most important contributions for SDRE control [4] is the Linear Quadratic Regulation (LQR). LQR is an optimal controller [3] minimizing a quadratic function cost given by

$$J_{LQR} = \frac{1}{2}\int_{t_0}^{\infty} (x^T Q x + u^T R x)\, dt \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control signal, and, $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times m}$ are the weight matrices semi-defined positive and defined positive respectively.

The idea of this function is making a trade off between performances using the Q weight to regulate the behavior of the states x and energy saving using the R weight to regulate the control signal u. The SDRE approach is an extension of the LQR controller: it is based on the same quadratic cost function Eq. (1) with the difference that weights Q and R can be state dependent:

$$J_{SDRE} = \frac{1}{2}\int_{t_0}^{\infty} (x^T Q(x) x + u^T R(x) x)\, dt \tag{2}$$

To solve this optimization problem, it is needed to define the specific problem in order to get constraints of the cost function. There are two kinds of constraints: the model and initial conditions. It can be written as:

$$\dot{x} = f(x) + g(x)u, \quad x(t_0) = x_0 \tag{3}$$

Applying a direct parameterization to transform the non-linear system of Eq. (3) into State Dependent Coefficients (SDC) representation [2], the dynamic equation of the system with control can be written in the form

$$\dot{x} = A(x)x + B(x)u \tag{4}$$

A is not unique. In fact there are an infinite number of parameterizations for SDC representation when the dynamic is non-linear. For the sake of example, let the dynamic function be a simple scalar function as

$$f(x_1, x_2) = 3x_1x_2 \tag{5}$$

Then A can take an infinite number of different forms as

$$\begin{aligned} A(x_1,x_2)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 3x_2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ A(x_1,x_2)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 0 & 3x_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ A(x_1,x_2)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 2x_2 & x_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned} \tag{6}$$

For multivariable case, it exists always at least two parameterizations A_1 and A_2 for all $0 \le \alpha \le 1$ satisfying

$$A(x) = \alpha A_1(x) + (1-\alpha)A_2(x) \tag{7}$$

Then $A(x, \alpha)$ represents the infinite family of SDC parameterization. The non-uniqueness of SDC parameterization creates additional degrees of freedom. The choice of parameterizations must be made in accordance with the control system of interest. However, this choice should not violate the controllability of the system, i.e., the state dependent controllability matrix must be full rank [8].

$$C_o(x) = \begin{bmatrix} B(x) & A(x)B(x) & \dots & A(x)^{n-1}B(x) \end{bmatrix} \tag{8}$$

The State Dependent Algebraic Riccati Equation (SDARE) can be obtained by applying the conditions for optimality of the variation calculus. In order to simplify expressions, state dependent matrix are sometimes written without reference to the states x: i.e. $A(x) \equiv A$. As a result, the Hamiltonian for the optimal control problem Eqs. (2) and (4) is

$$H(x, u, \lambda) = \frac{1}{2}(x^T Q x + u^T R u) + \lambda(Ax + Bu) \tag{9}$$

where $\lambda \in \mathbb{R}^n$ is the Lagrange multiplier.

Applying to Eq. (9) the necessary conditions for the optimal control given by $\dot{\lambda} = -\frac{\partial H}{\partial x}$, $\dot{x} = -\frac{\partial H}{\partial \lambda}$ and $0 = -\frac{\partial H}{\partial x}$ leads to

$$\begin{aligned} \dot{\lambda} = &-Qx - \frac{1}{2}x^T\frac{\partial Q}{\partial x}x - \frac{1}{2}u^T\frac{\partial R}{\partial x}u \\ &-\left[\frac{\partial Ax}{\partial x}\right]^T\lambda - \left[\frac{\partial Bu}{\partial x}\right]^T\lambda \end{aligned} \tag{10}$$

$$\dot{x} = A(x)x + B(x)u \tag{11}$$

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