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Experimental validation of wavelet based solution for dynamic response of railway track subjected to a moving train

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A B S T R A C T

New approaches allowing effective analysis of railway structures dynamic behaviour are needed for appropriate modelling and understanding of phenomena associated with train transportation. The literature highlights the fact that nonlinear assumptions are of importance in dynamic analysis of railway tracks. This paper presents wavelet based semi-analytical solution for the infinite Euler–Bernoulli beam resting on a nonlinear foundation and subjected to a set of moving forces, being representation of railway track with moving train, along with its preliminary experimental validation. It is shown that this model, although very simplified, with an assumption of viscous damping of foundation, can be considered as a good enough approximation of realistic structures behaviour. The steady-state response of the beam is obtained by applying the Galilean co-ordinate system and the Adomian's decomposition method combined with coiflet based approximation, leading to analytical estimation of transverse displacements. The applied approach, using parameters taken from real measurements carried out on the Polish Railways network for fast train Pendolino EMU-250, shows ability of the proposed method to analyse parametrically dynamic systems associated with transportation. The obtained results are in accordance with measurement data in wide range of physical parameters, which can be treated as a validation of the developed wavelet based approach. The conducted investigation is supplemented by several numerical examples.

1. Introduction

Dynamic systems referring to realistic structures, especially those appearing in railway infrastructure, still need new methods allowing their parametrical analysis. Mathematical models along with effective procedures leading to the analysis of dynamic behaviour of railway tracks can replace laborious and also very often highly doubtful numerical computations usually used in the investigation of such structures [1]. Problems related to moving load analysis are widely discussed in the literature [2]. However, nonlinear properties or stochastic variations of physical characteristics are still considered as open problems mainly due to a lack of appropriate tools allowing to obtain analytical solutions [3–6]. This paper discusses developed previously semi-analytical method based on Adomian's decomposition supported by coiflet expansion of functions representing the system response [7–9]. Although some important results are already obtained by using this method [4,5,9–16], the approach itself still needs extensive investigations in order to recognize its advantages. On the other hand it needs experimental validation showing its ability to represent real

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scenarios [17,18]. It is shown in this paper, that even relatively simple model consisting of one Euler–Bernoulli beam representing the rail can reflect real behaviour of rail track when using parameters taken from experiments and analytical solution obtained by applying the developed wavelet based method [7,8]. Measurements taken for the Polish Railways network during passage of fast train Pendolino EMU-250 are used as reference results showing accordance with wavelet-based simulations in wide range of physical parameters [18].

2. Model formulation

The dynamic nonlinear equation for the Euler–Bernoulli beam can be written in the following form:

$$EI\frac{\partial^4 w(\tilde{x},t)}{\partial \tilde{x}^4}+\rho_B\frac{\partial^2 w(\tilde{x},t)}{\partial t^2}+c\frac{\partial w(\tilde{x},t)}{\partial t}+k_L w(\tilde{x},t)+k_N w^3(\tilde{x},t)=P(\tilde{x},t) \tag{1}$$

where $w(\tilde{x},t)$ is the vibration of the beam, EI is the bending stiffness, $\rho_B=\rho A$ is the linear density of the beam, A is the cross-section area of the beam, c is the viscous damping of foundation, k_L is the linear coefficient of foundation stiffness and k_N is the nonlinear part of foundation stiffness. $P(\tilde{x},t)$ represents the vertical moving load consisting of a set of four loads related to wheels of train:

$$P(\tilde{x},t)=P_S(\tilde{x},t)+P_D(\tilde{x},t)=\sum_{l=0}^{3}\left(\frac{P_0}{a}+\frac{\Delta P}{a}e^{i\tilde{\omega}t}\right)\cos^2\left(\frac{\pi(\tilde{x}-Vt-s_l)}{2a}\right)H\left(a^2-(\tilde{x}-Vt-s_l)^2\right) \tag{2}$$

$$P_S(\tilde{x},t)=\sum_{l=0}^{3}\frac{P_0}{a}\cos^2\left(\frac{\pi(\tilde{x}-Vt-s_l)}{2a}\right)H(a^2-(\tilde{x}-Vt-s_l)^2) \tag{3a}$$

$$P_D(\tilde{x},t)=\sum_{l=0}^{3}\frac{\Delta P}{a}e^{i\tilde{\omega}t}\cos^2\left(\frac{\pi(\tilde{x}-Vt-s_l)}{2a}\right)H\left(a^2-(\tilde{x}-Vt-s_l)^2\right) \tag{3b}$$

where $H(.)$, $2a$, s_l, $\tilde{\omega}$ and V are the Heaviside function, the span of each load, the distance between separated consecutive loads (the distance of next wheels from the first considered train axle), the frequency and the velocity of the moving load, respectively. Each term includes a stationary force P_0 and a harmonically varying part representing variations produced by track irregularities. $\Delta P=0.15P_0$ is an amplitude of additional harmonically varying force that can be related to irregularities appearing on rail. 15% of the static load is the amount of force determined experimentally. It was estimated that the static deflection amplitude of unsupported rail between two sleepers is around 11 μm. This value, along with assumption that the contact spring between wheel and rail is linear, gives an estimated value of ΔP being a product of these two parameters. The value of the contact spring is chosen on the basis of quantities given in [19,20]. It is assumed in this paper that the rail head has irregularities with length of $L=0.5$ m, placed directly side by side, that is with no space between them. The amplitude of this rail imperfection is constant and the shape of rail surface has cosine form. Therefore one should take into account the angular phase associated with different positions of wheels at fixed time. This additional factor influencing the dynamical part of load is considered in parametrical studies. It is assumed that the form of each single force depends on position of wheel on the rail surface (irregularity) at fixed time. This leads to the following representation of the load frequency:

$$\tilde{\omega}=\Omega+\phi_l/t, \tag{4}$$

where $\Omega=2\pi V/L$ and $\phi_l=\frac{2\pi\Phi_l}{L}$ is the phase shift associated with the wheel position on the cosine shape of irregularity. The parameter Φ_l can be treated as an angular phase. This kind of the load representation is an important novelty and it was not presented in the literature so far. Numerical simulations show that the phase shift should be considered for the rail track response caused by moving train. This assumption does not reduce the effectiveness of nonlinear solution procedure. One can say that simulations performed so far show significant differences when one considers the angular phase as additional factor influencing the dynamical part of moving load for some systems of physical parameters, especially when higher number of wheels (loads) is taken into account. Numerical simulations also show that noticeable changes appear when nonlinear properties of the model are taken into account and, which is even more important, different frequencies of the loads produced by each wheel separately are modelled. This corresponds to a number of imperfections appearing on the head of rail. It is assumed in this paper that other rail imperfections, different than the one described above, do not influence track vibrations and they might cause very slight change of the load axle only. In addition, the rail deflection between sleepers, although considered in reality as one of factors producing dynamic harmonic changes of the load [21], is omitted. The amplitude ΔP and the form of imperfection do not depend on train velocity and vehicle parameters. The frequency characteristics of the system response does not depend on the load parameters either.

To have a more realistic representation of the load, all the mentioned factors should be modelled: different rail irregularities, rail deflection between sleepers, a wide spectrum of frequencies and also kind of independency between wheels, unless the whole train is described along with its multibody interactions.

The solution method applied in this paper is based on the Adomian's decomposition [7,9,22] combined with wavelet approximation of functions in L^2 space [8,9]. The most classical representation of the Adomian polynomials and so called