Mechanical Systems and Signal Processing 79 (2016) 192–202

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Vibration band gaps for elastic metamaterial rods using wave finite element method

E.D. Nobrega [a,*], F. Gautier [b], A. Pelat [b], J.M.C. Dos Santos [a]

[a] University of Campinas, UNICAMP-FEM-DMC, Rua Mendeleyev, 200, CEP 13083-970 Campinas, SP, Brazil
[b] Université du Maine, ENSIM - LAUM, 1 Rue Aristote, 72000 Le Mans, France

ARTICLE INFO

Article history:
Received 25 February 2016
Accepted 27 February 2016
Available online 15 March 2016

Keywords:
Elastic metamaterial
Periodic structure
Wave propagation
WFEM

ABSTRACT

Band gaps in elastic metamaterial rods with spatial periodic distribution and periodically attached local resonators are investigated. New techniques to analyze metamaterial systems are using a combination of analytical or numerical method with wave propagation. One of them, called here wave spectral element method (WSEM), consists of combining the spectral element method (SEM) with Floquet–Bloch's theorem. A modern methodology called wave finite element method (WFEM), developed to calculate dynamic behavior in periodic acoustic and structural systems, utilizes a similar approach where SEM is substituted by the conventional finite element method (FEM). In this paper, it is proposed to use WFEM to calculate band gaps in elastic metamaterial rods with spatial periodic distribution and periodically attached local resonators of multi-degree-of-freedom (M-DOF). Simulated examples with band gaps generated by Bragg scattering and local resonators are calculated by WFEM and verified with WSEM, which is used as a reference method. Results are presented in the form of attenuation constant, vibration transmittance and frequency response function (FRF). For all cases, WFEM and WSEM results are in agreement, provided that the number of elements used in WFEM is sufficient to convergence. An experimental test was conducted with a real elastic metamaterial rod, manufactured with plastic in a 3D printer, without local resonance-type effect. The experimental results for the metamaterial rod with band gaps generated by Bragg scattering are compared with the simulated ones. Both numerical methods (WSEM and WFEM) can localize the band gap position and width very close to the experimental results. A hybrid approach combining WFEM with the commercial finite element software ANSYS is proposed to model complex metamaterial systems. Two examples illustrating its efficiency and accuracy to model an elastic metamaterial rod unit-cell using 1D simple rod element and 3D solid element are demonstrated and the results present good approximation to the experimental data.

1. Introduction

The engineering studies of periodic structures using wave propagation began to spread in the mid-1970s with Mead's works [1–4]. In recent decades, new methods have been developed that use the same basic concepts of periodicity together

* Corresponding author. Tel.: +55 1935213388.
E-mail address: edilsonnobre@gmail.com (E.D. Nobrega).

http://dx.doi.org/10.1016/j.ymssp.2016.02.059
0888-3270/© 2016 Elsevier Ltd. All rights reserved.

with approximated solution as a way to reduce computational costs and to solve complex engineering models that cannot be solved analytically neither numerically using traditional methods [5–8]. One of them is the wave finite element method (WFEM), which consists of modeling a small slice of elastic waveguide by FEM, to apply periodicity condition with Floquet–Bloch's theorem to obtain a transfer matrix eigenproblem. The solution provides attenuation and wave-modes, from where wave motion amplitudes are obtained. The method has been applied in various types of finite element model, such as beams, thin plates, cylindrical shells, including different material properties, couplings and mediums. More recently, it has been applied as a numerical solution for modeling periodic structures in the mid-frequency range where conventional methods are not efficient.

In the last decade applied research in phononic materials has been plentiful. However, some fundamental evaluation from engineering point of view, such as model simulation (analytical, numerical and hybrid) and experimental approaches developed for conventional materials and structures, needs to be employed to understanding and explore the metamaterial systems [9]. One of the most attractive characteristics of acoustic and elastic metamaterial is their wave filtering behavior. This provides some frequency ranges known as band gaps or forbidden bands where the waves cannot propagate. Band gaps are generated on spatial periodicity of the impedance mismatch domains which produces Bragg scattering effect. Also, locally resonant mechanism [10] provides band gaps at sub-wavelength, which are well below the Bragg scattering band gaps.

In 2006, a theoretical and experimental study of longitudinal wave propagation in a rod structure including periodic local resonators was presented by Wang et al. [11]. They shown that both results produce an asymmetric band gap attenuation which is influenced by local resonator stiffness and mass ratios. Nevertheless, this work was centered on the band gaps conception and its property to attenuate vibration, without exploring fully the band gap formation mechanisms.

More late, a theoretical and numerical study of locally resonant elastic metamaterial rod system with periodic multi-degree-of-freedom (M-DOF) resonators was presented by Xiao et al. [12]. The band gap behavior and vibration attenuation performance was analyzed in a more systematic way. A new metamaterial rod model based on a combination of spectral element method and Floquet–Bloch's theorem was proposed, which will be called here wave spectral element method (WSEM). They provided explicit expressions to predict band edge frequencies, demonstrated that both Bragg- and resonance-type band gaps co-exist in metamaterial rods and that multiple resonance band gaps can be achieved using M-DOF local resonators. These authors had also applied WSEM for band gap investigation in flexural metamaterial beams with local resonators [13].

Recently, Khajehtourian and Hussein [14] presented a study of wave dispersion in a nonlinear elastic metamaterial rod system with periodically attached local resonators. The type of nonlinearity considered is large elastic deformation. The metamaterial rod model is based on a combination of standard transfer matrix method and Floquet–Bloch's theorem. The results demonstrate that nonlinearities on metamaterial rods can affect band gap position, width, and its type (Bragg scattering or local resonance). They shown that large deformation alone may induce a pair of Bragg- and resonance-type band gaps to merge in a one hybrid and form a combined wide band gap. They also shown that as the wave amplitude increases, the effect of the nonlinearity in the metamaterial rod system is no longer negligible and the error incurred by assuming linear elastic wave propagation theory increases quickly.

In this paper, the wave finite element method (WFEM) is applied to analyze elastic metamaterial rods with spatial periodic distribution and periodically attached M-DOFs local resonators. The main purpose of this paper is to demonstrate the accuracy and efficiency of WFEM for modeling metamaterial system. Considering that WSEM is based on the exact analytical solution of rod govern equation and WFEM is based on the approximated finite element solution, it can be said that one spectral element must be equal to infinite elements of FEM. It means that for a uniform metamaterial rod calculated by WSEM the unit-cell dynamic stiffness matrix requires only one dynamic spectral element matrix, while by WFEM the unit-cell dynamic stiffness matrix requires an assembly of as many FEM stiffness and mass element matrices as required to obtain good convergence. Therefore, WSEM will be used here as a reference method to verify the accuracy of WFEM. Clearly, for the uniform metamaterial rod considered in the paper, WSEM is more accurate than WFEM. Nevertheless, WFEM is able to model more complex metamaterial rod system, such as nonuniform rods, which cannot be easily modeled using WSEM. Both methods are computationally implemented, evaluated and compared to each other.

By using an actual elastic metamaterial waveguide an experimental test was performed. The elastic metamaterial waveguide was originally developed to be used in another research project as a beam-like (flexural waves) and a shaft-like (torsional waves) structure with spatial periodic distribution and local resonators. For this work it was configured as a rod-like (longitudinal waves) structure, which maintain the spatial periodic distribution, but the local resonators becomes inactive. The metamaterial rod is manufactured with plastic (Vero White Plus) in a 3D printer with UV curing technology. All simulated results with WSEM and WFEM are compared with the experimental data and some different behaviors and mismatches between numerical and experimental FRFs were found. After a trial-and-error numerical model updating by varying material property parameters (Young modulus, mass density and internal damping) these differences are minimized, and the numerical models can localize the band gap position and width very close to the experimental results.

A hybrid approach combining WFEM with a finite element analysis commercial software ANSYS (Mechanical APDL Release 14.5) is proposed and two examples illustrating its efficiency to model an elastic metamaterial rod unit-cell using 1D and 3D FEM models are demonstrated. Based on these results it can be extrapolated that WFEM, as compared with WSEM and other analytical and numerical methods, is able to model metamaterial unit-cell using commercial FEM packages, which facilitates greatly its application for real complex engineering structures.