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Chaotic dynamics of flexible beams driven by external white noise

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Mathematical models of continuous structural members (beams, plates and shells) subjected to an external additive white noise are studied. The structural members are considered as systems with infinite number of degrees of freedom. We show that in mechanical structural systems external noise can not only lead to quantitative changes in the system dynamics (that is obvious), but also cause the qualitative, and sometimes surprising changes in the vibration regimes. Furthermore, we show that scenarios of the transition from regular to chaotic regimes quantified by Fast Fourier Transform (FFT) can lead to erroneous conclusions, and a support of the wavelet analysis is needed. We have detected and illustrated the modifications of classical three scenarios of transition from regular vibrations to deterministic chaos. The carried out numerical experiment shows that the white noise lowers the threshold for transition into spatio-temporal chaotic dynamics. A transition into chaos via the proposed modified scenarios developed in this work is sensitive to small noise and significantly reduces occurrence of periodic vibrations. Increase of noise intensity yields decrease of the duration of the laminar signal range, i.e., time between two successive turbulent bursts decreases. Scenario of transition into chaos of the studied mechanical structures essentially depends on the control parameters, and it can be different in different zones of the constructed charts (control parameter planes). Furthermore, we found an interesting phenomenon, when increase of the noise intensity yields surprisingly the vibrational characteristics with a lack of noisy effect (chaos is destroyed by noise and windows of periodicity appear).

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1. Introduction

It is well known that in deterministic physical systems there are several scenarios of transition from regular to chaotic oscillations. Ruelle and Takens [1] have shown that chaos can be achieved via combination of only three frequencies. "Noisy" behavior in this scenario is associated with a strange attractor arisen after three consecutive Hopf bifurcations [2]. Simple deterministic systems are able to generate internal chaotic response. It can be realized via a sequence of Hopf bifurcations, and the period doubling scenario through intermittency [2–4]. On the other hand, a random nature of the input may induce much richer variety of non-linear phenomena than those regarding purely deterministic cases. It is expected that noisy transitions from regular to chaotic dynamics exhibited by structural members are similar to the phase transitions, and to the transitions occurring in non-equilibrium systems with deterministic external inputs. We are aimed to show that it is possible to extend the classical transition scenarios and associated non-linear phenomena to physical systems in which the noise plays an important role. Thus, it becomes possible to carry out the theoretical study of noisy induced transitions into chaotic regimes of the studied continuous mechanical systems. Theory of deterministic multidimensional systems has been already introduced in references [5–15]. The present work aims to apply and extend these studies in the event of noise-induced transitions.

Though a study of chaotic vibrations of structural systems attracted a large attention within community of applied mathematicians and engineers usually, the problem has been strongly reduced to that of non-linear dynamics of either one- or two-degrees-of-freedom lumped systems. For example, fluctuations in distributed systems are often replaced by a study of the single-mode non-linear vibrations of Bernoulli–Euler beams, taking into account the geometric nonlinearity. The results of these studies should be considered as qualitative, since the increase in the number of modes often leads to substantially different non-linear dynamics. In reference [16] global bifurcations and chaotic dynamics in nonlinear plane vibrations of a cantilever beam under axial harmonic excitation and transverse excitations at the free end of the beam are studied. Finite element method for the study of forced nonlinear oscillations of the beam has been applied in work [17]. Chaos exhibited by vibrations of plates and shells with geometric nonlinearities has been extensively analyzed in references [5–13]. In the case of beams, the new stochastic linearization method has been applied to investigate the non-linear mean square response of a beam under time-dependent stationary random excitation [18]. It has been shown, that the proposed technique can yield more accurate results for the mean square response of the beam in comparison to the standard stochastic linearization approach. Dahlberg [19] has applied the model analysis to study the influence of modal cross-spectral densities versus the spectral densities of simply supported beams. The response power spectral density and mean-square response have been used by Jacquot [20] while studying beam structures excited by a stationary random process. The effect of axial loads on transverse vibrations of an Euler–Bernoulli clamped-pinned beam under random vibration has been analyzed in reference [21]. The concepts of the moment Lyapunov exponent and the Lyapunov exponent of a Timoshenko beam under bounded noise excitation have been used in reference [22]. Both almost-sure stability and moment stability of the stationary solutions of the elastic beam subjected to the stochastic axial load have been investigated. The so called shape memory alloy beams non-linear dynamics have been studied including bifurcation and chaotic phenomena [23–25]. The superelastic shape memory alloy systems under random stationary excitations have been analyzed in reference [26]. Recently, the shape memory alloy beam under narrow band noise excitations (harmonic function with constant amplitude and random frequency) has been investigated by Ge [27]. However, a strong Galerkin-based truncation of the governing PDEs reduced the consideration to non-linear one-degree-of freedom mechanical systems. The electromechanical response of the pre-buckled inverted cantilevered beam subjected to a combination of harmonic and broadband random excitation has been studied by Friswell et al. [28]. Since the pre-buckled beam stiffness is low, the displacement response yielded multiple solutions being exploited in the harvesting device. The amplitude of random noise excitation, where the harvester is unable to sustain the high amplitude solution, has been investigated and validated experimentally.

In the case of plates, active control of noise radiation from vibrating plate excited by a harmonic line moment has been proposed by Lee and Chen [29]. The control has been achieved by various configurations of piezoelectric actuators and the optimal control unit voltage has been utilized. It has been shown, among other, that the model suppression and modal restructuring play a key role in radiated power attenuation. Numerical prediction of noise transmission loss through sandwich plates subjected to an acoustic plane wave or a diffuse sound field excitation has been reported by Assaf and Guerich [30]. The diffuse sound field has been modeled as a superposition of uncorrelated plane waves with equal amplitude, whereas the vibroacoustic equations have been discretized by a triangular finite element. A stochastic non-linear model has been proposed to describe vibrations of a rectangular thin plate under axial inplane excitation with random environment factors, and the stochastic Hopf bifurcation of the vibration model has been investigated [31]. Wiciak [32] has investigated reduction of plate vibrations and radiated noise by using piezoelectric actuators in an asymmetric configuration. In particular, influence of the actuators activation and shape form on the plate response has been studied, and the experimental results have been compared with the numerical approach based on the finite element method. Both motion and sound of a thin elastic plate subjected to uniform low-Mach flow and actuated at its leading edge, has been studied by Manela [33]. Periodic and non-periodic actuations have been investigated.

In the case of shells we report only a few works. White [34] investigated the transmission of an acoustic wave through an infinite, nonhomogeneous closed cylindrical shell. It has been shown that the presence of stiffening corrugations and irregularities in the shell leads to a random-vibration field, and hence the transmission of random sound through flat panels can be used for noise-reduction estimates. Durant et al. [35] established a comparison between the measured and the