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Uncertainty quantification in operational modal analysis with stochastic subspace identification: Validation and applications

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ABSTRACT

Identified modal characteristics are often used as a basis for the calibration and validation of dynamic structural models, for structural control, for structural health monitoring, etc. It is therefore important to know their accuracy. In this article, a method for estimating the (co) variance of modal characteristics that are identified with the stochastic subspace identification method is validated for two civil engineering structures. The first structure is a damaged prestressed concrete bridge for which acceleration and dynamic strain data were measured in 36 different setups. The second structure is a mid-rise building for which acceleration data were measured in 10 different setups. There is a good quantitative agreement between the predicted levels of uncertainty and the observed variability of the eigenfrequencies and damping ratios between the different setups. The method can therefore be used with confidence for quantifying the uncertainty of the identified modal characteristics, also when some or all of them are estimated from a single batch of vibration data. Furthermore, the method is seen to yield valuable insight in the variability of the estimation accuracy from mode to mode and from setup to setup: the more informative a setup is regarding an estimated modal characteristic, the smaller is the estimated variance.

1. Introduction

The problem of quantifying the uncertainty of modal characteristics that are estimated from a single batch of (operational) vibration data gained interest during the past few years, with the development of expressions for the variance of the estimates obtained from two high-performance system identification methods: stochastic subspace identification [1] and maximum likelihood estimation [2]. These expressions enable the estimation of the variance of the eigenfrequencies, damping ratios, and mode shapes, obtained from a single batch of data, as well as the covariance between them. They also allow computing confidence intervals because the estimates of the modal characteristics are asymptotically normally distributed.

In this article, the variance estimation procedure is validated for the stochastic subspace identification (SSI) method, which has become a standard for operational modal analysis [3–5]. SSI essentially estimates a state-space model from an observed output correlation sequence using linear algebra techniques. An eigenvalue decomposition of the identified state-space model then yields the eigenfrequencies, damping ratios and mode shapes. Several algorithmic variants of SSI exist, amongst which SSI-cov (covariance-driven stochastic subspace identification) [6,7] and NExT-ERA (eigensystem realization algorithm combined with the natural excitation technique) [8–10] are very commonly employed in operational modal analysis. These algorithms are all specific implementations of a single basic algorithm [11] and their computational and statistical performance is largely similar in practice [3,5].

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The SSI method has the particular advantage of combining a high estimation accuracy with a high computational robustness and efficiency. Optimality of the estimation accuracy in the sense of asymptotic minimum variance (also called statistical efficiency) has also been theoretically proven for some implementations of SSI [12]. The accuracy of SSI estimation is therefore comparable to that of maximum likelihood (ML) estimation, for which asymptotic statistical efficiency is guaranteed [13–15]. However, ML estimation is computationally costly because it involves the iterative minimization of a non-convex objective function. Furthermore, simulations have shown that the method can get stuck in local minima even for starting values that are very close to the exact values [5]. In order to reduce the computation cost, an approximate ML estimation is usually performed in which the covariance between the different output estimates is disregarded [16]. Note that the Bayesian modal identification algorithms that have been recently proposed essentially reduce to maximum likelihood estimation since prior information is not employed [17]. Expressions for the variance of the estimated modal characteristics also exist for the p-LSCF method (poly-reference least squares complex frequency domain) [18]. This method results in clear stabilization diagrams which facilitate the modal identification process, but it can also result in biased estimates, especially of the damping ratio [5], and therefore the variance alone may largely underestimate the total statistical uncertainty of the modal characteristics that are obtained with this method.

The covariance estimation procedure for the modal characteristics (or state-space model parameters) estimated with SSI is based on a first-order sensitivity analysis of the identified values to the output correlations where the algorithm starts from. This sensitivity analysis, the related covariance expressions, and a numerical verification have been presented in [1]. An implementation of the covariance expressions that is optimized in terms of computational efficiency was presented in [19], and an extension towards the joint analysis of multi-setup measurements was presented in [20].

In the present article, the method for estimating the (co)variance of modal characteristics that are identified with SSI is validated for two civil engineering structures: a damaged prestressed concrete bridge for which acceleration and dynamic strain data were measured in 36 different setups, and a mid-rise building for which acceleration data were measured in 10 different setups. For each structure, the validation consists of two stages. In the first stage, the estimated uncertainty of the identified eigenfrequencies and damping ratios is compared with the observed variability across all setups. The observed variance (or sample variance) and the predicted variance should be the same if the amount of information contained in the data is the same in all setups. This can reasonably be expected when the measurements for all setups are carried out in similar circumstances. The second stage focuses on the differences in estimation accuracy between different modes and different setups: if a particular experiment is less informative on a first mode than it is on a second mode, e.g. because the first mode is less well excited, then the predicted uncertainty should be larger for the first mode than for the second mode.

The remainder of this article is organized as follows. In Section 2, the estimation of modal characteristics with SSI and the computation of the variance of these estimates are briefly summarized. Section 3 contains the first detailed validation study, which involves the ambient modal testing of a damaged prestressed concrete bridge. The second validation study, comprising the identification of the modal characteristics of a mid-rise building under ambient wind excitation, is discussed in Section 4. The final conclusions are presented in Section 5.

2. Stochastic subspace identification with uncertainty quantification

As indicated previously, several algorithmic variants of SSI exist. The following analysis concentrates on the SSI-cov algorithm because of two reasons: the algorithm is simple and easy to implement, and it is also theoretically optimal in the sense that the resulting system description is deterministically balanced [21]. The SSI-cov algorithm is also known under the name principal component (PC) algorithm [11].

2.1. System model

The input–output relationship of a linear time-invariant system can, after discretization in time, be described with the following state-space model:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k \tag{1}$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k + \mathbf{e}_k, \tag{2}$$

where $\mathbf{x}_k \in \mathbb{R}^n$ is the state of the system, $\mathbf{y}_k \in \mathbb{R}^{n_y}$ is the output vector, $\mathbf{u}_k \in \mathbb{R}^{n_u}$ is the input vector, and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are the system matrices. In operational modal analysis, only the outputs $\mathbf{y}_k$ are observed, with measurement error $\mathbf{e}_k$. In general, little information on the loading $\mathbf{u}_k$ and the measurement error $\mathbf{e}_k$ is available, other than that they take finite values that are centered around zero. For this reason $\mathbf{u}_k$ and $\mathbf{e}_k$ are modeled as white noise random processes. This is a good assumption as long as the ambient excitation spectrum is not dominated by specific frequency components, which is for instance the case when strong harmonics are present in the excitation [5]. The white noise model also follows naturally from a Bayesian perspective, because it represents a state of maximum information entropy (or maximum uncertainty) given the available information [22].

Since $\mathbf{u}_k$ and $\mathbf{e}_k$ are assumed to be discrete white noise random processes, the state-space model can be reformulated as

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{w}_k \tag{3}$$