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Fundamental two-stage formulation for Bayesian system identification, Part II: Application to ambient vibration data

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ABSTRACT

A fundamental theory has been developed for a general two-stage Bayesian system identification problem in the companion paper (Part I). This paper applies the theory to the particular case of structural system identification using ambient vibration data. In Stage I, the modal properties are identified using the Fast Bayesian FFT method. Given the data, their posterior distribution can be well approximated by a Gaussian distribution whose mean and covariance matrix can be computed efficiently. In Stage II, the structural model parameters (e.g., stiffness, mass) are identified incorporating the posterior distribution of the natural frequencies and mode shapes in Stage I and their conditional distribution based on the theoretical structural finite element model. Synthetic and experimental data are used to illustrate the proposed theory and applications. A number of factors commonly relevant to structural system identification are studied, including the number of measured degrees of freedom, the number of identifiable modes and sensor alignment error.

1. Introduction

A general theory has been presented in the companion paper [1] for a two-stage Bayesian system identification problem. It fundamentally expresses the posterior probability density function (PDF) of structural model parameters in Stage II in terms of the posterior PDF of the modal parameters in Stage I. In this paper, the theory is applied to the identification of structural model parameters (e.g., stiffness, mass), which is the problem originally motivated the development of the general theory. The data is assumed to consist of digital acceleration time histories measured at a limited number of degrees of freedom (dofs) of the subject structure under ambient environment. The loading is unknown but assumed to be broadband random within the resonance band of the identified modes. This context is of high relevance in practice, as ambient vibration tests are becoming economically viable and commercially sustainable [2,3]. It is also of high scientific relevance because the identification uncertainty of modal parameters based on (output-only) ambient data is often significantly higher than their counterparts identified from properly managed free or forced vibration data. As mentioned in the companion paper, different variants of two-stage Bayesian formulations for structural system identification have been proposed, e.g. [4–9], although they all involve heuristics in the formulation of the likelihood function in Stage II.

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For clarity we first give an overview of the two-stage approach applied to structural identification problem in the context of the theoretical framework developed in the companion paper. Using ambient vibration data, the objective is to identify the set of structural model parameters $\boldsymbol{\theta}$ involved in the characterization of the finite element model of the real structure, e.g., stiffness, mass, boundary conditions, etc. In Stage I, Fast Bayesian FFT (Fast Fourier Transform) method is used for identifying the modal properties based on ambient vibration data [10–13], see a recent review in [14]. The method is well-suited for ambient modal identification for its computational efficiency and assumption robustness. Operating in the frequency domain, the data D effectively consists of the FFT of the measured acceleration time histories within the resonance frequency bands of the modes selected by the analyst. As far as structural system identification is concerned, the information content of this FFT data is equivalent to the original time domain data, because the FFTs in other frequency bands are irrelevant or difficult to model. Using only the FFT data in the selected frequency bands for identification significantly reduces the number of modal parameters to be identified simultaneously and requires minimal assumption on the ambient excitation.

The full set of modal parameters $\boldsymbol{\alpha}$, from which an explicit likelihood function $p(D|\boldsymbol{\alpha})$ can be derived, comprises the natural frequencies, damping ratios, partial mode shapes (i.e., confined to the measured dofs), the power spectral density (PSD) matrix of the modal forces and the PSD of the prediction error (arising from, e.g., sensor noise). With sufficient data, these parameters are globally identifiable and their posterior PDF can be well approximated by a Gaussian distribution with mean and covariance matrix that can be computed efficiently. Uniform (i.e., constant) prior distributions are used in practice for modal identification problems. As a result, the hypothetical posterior PDF $p_0(\boldsymbol{\alpha}|D)$ and the actual posterior PDF $p(\boldsymbol{\alpha}|D)$ are identical. Within $\boldsymbol{\alpha} = [\varpi, \boldsymbol{\upsilon}]$, the set ϖ for identifying the structural parameters in Stage II comprises the natural frequencies and partial mode shapes because they can be theoretically predicted by a structural (e.g., finite element) model. The set $\boldsymbol{\upsilon}$ comprises the remaining modal parameters, i.e., the damping ratios, PSD matrix of modal forces and the PSD of prediction error. As a property of Gaussian distribution, the marginal distribution $p_0(\varpi|D)$ is also Gaussian, whose mean and covariance matrix can be directly taken from those of the full distribution $p_0(\boldsymbol{\alpha}|D)$.

This work focuses on the case when there is no structural prediction error. That is, the natural frequencies and mode shapes can be completely determined by the structural parameters so that $p(\varpi|\boldsymbol{\theta}) = \delta(\varpi - \tilde{\varpi}(\boldsymbol{\theta}))$ is a Dirac-Delta function centered at the theoretical structural model prediction $\tilde{\varpi}(\boldsymbol{\theta})$. This scope is considered as it is consistent with the conventional scenario studied in the literature, providing a starting point for applying the general theory. Modeling $p(\varpi|\boldsymbol{\theta})$ in a non-trivial manner and incorporating its information for updating $\boldsymbol{\theta}$ requires substantially more consideration that deserves a separate line of research.

This paper is organized as follow. The structural modeling assumptions are first described, followed by an outline of Fast Bayesian FFT method in Stage I. Theoretical and computational issues in Stage II are discussed. A comparison with the conventional formulations is then given, followed by a summary of the whole procedure. Illustrative examples with synthetic and experimental data are presented to verify the method with applications.

2. Problem context

Consider a linear elastic structure, modeled by the conventional structural dynamics equation

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{W}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, and $\mathbf{W}$ are the mass matrix, damping matrix, stiffness matrix and force vector, respectively. Assuming classical damping, the response can be expressed as a sum of modal contributions:

$$\mathbf{x}(t) = \sum_i \mathbf{u}_i \eta_i(t) \tag{2}$$

where $\mathbf{u}_i$ and η_i are respectively the full mode shape and modal response of the i-th mode; the sum is overall all modes of the structure. The full mode shape $\mathbf{u}_i$ satisfies the generalized eigenvalue equation:

$$\mathbf{K}\mathbf{u}_i = \omega_i^2 \mathbf{M}\mathbf{u}_i \tag{3}$$

where $\omega_i = 2\pi f_i$ and f_i are the natural frequency in rad/s and in Hz, respectively. The modal response η_i satisfies the uncoupled modal equation of motion:

$$\ddot{\eta}_i(t) + 2\zeta_i\omega_i\dot{\eta}_i(t) + \omega_i^2\eta_i(t) = w_i(t) \tag{4}$$

where

$$w_i(t) = \frac{\mathbf{u}^T\mathbf{W}(t)}{\mathbf{u}^T\mathbf{M}\mathbf{u}} \tag{5}$$

is the modal force.

The goal of the structural identification problem in this paper is to identify the structural parameters $\boldsymbol{\theta}$ from ambient vibration data of the as-built structure at a limited number of dofs. Only the stiffness matrix $\mathbf{K}$ and the mass matrix $\mathbf{M}$ are assumed to possibly depend on $\boldsymbol{\theta}$. This assumption arises from practical consideration in structural engineering where there is no acceptable means for modeling the damping of real structures. The loading $\mathbf{W}(t)$ is not measured but is assumed to be