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Online updating and uncertainty quantification using nonstationary output-only measurement

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ABSTRACT

Extended Kalman filter (EKF) is widely adopted for state estimation and parametric identification of dynamical systems. In this algorithm, it is required to specify the covariance matrices of the process noise and measurement noise based on prior knowledge. However, improper assignment of these noise covariance matrices leads to unreliable estimation and misleading uncertainty estimation on the system state and model parameters. Furthermore, it may induce diverging estimation. To resolve these problems, we propose a Bayesian probabilistic algorithm for online estimation of the noise parameters which are used to characterize the noise covariance matrices. There are three major appealing features of the proposed approach. First, it resolves the divergence problem in the conventional usage of EKF due to improper choice of the noise covariance matrices. Second, the proposed approach ensures the reliability of the uncertainty quantification. Finally, since the noise parameters are allowed to be time-varying, nonstationary process noise and/or measurement noise are explicitly taken into account. Examples using stationary/nonstationary response of linear/nonlinear time-varying dynamical systems are presented to demonstrate the efficacy of the proposed approach. Furthermore, comparison with the conventional usage of EKF will be provided to reveal the necessity of the proposed approach for reliable model updating and uncertainty quantification.

1. Introduction

Bayesian inference provides a rigorous framework for parametric identification and uncertainty quantification [1–4]. It stimulates scientific investigations and various engineering applications, such as structural health monitoring [5,6],damage detection [7,8], model class selection [9,10], optimal sensor configuration design [11], geotechnical engineering [12], seismic attenuation relationships [13], sensitivity analysis [14], outlier detection [15], and molecular dynamics [16], etc. Bayesian filtering is a representative Bayesian inference technique for state-space analysis [17–20]. In particular, the well-known Kalman filter is one of the most widely applied recursive state estimation technique for linear dynamical systems [18,21,22]. Based on the mathematical foundation of the Kalman filter, the extended Kalman filter (EKF) was developed for nonlinear systems [23]. By introducing an augmented state vector, the EKF can be applied to parametric identification. Taking the advantage of Bayesian inference, the EKF provides not only the estimation of the parameters but also the associated

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uncertainty. As a result, the EKF has become a standard technique for state tracking, system identification and control design for dynamical systems [22–26].

Despite the widespread usage, there is a challenging issue in the implementation of EKF, i.e., to determine the covariance matrices of the process noise and the measurement noise. In practice, this is done in an ad hoc trial-and-error manner. However, improper choice of these matrices may lead to bias estimation, unreliable uncertainty estimation and even divergence problems. Previous studies [27–30] provided important insights into the divergence mechanisms, asymptotic behavior and estimation performance of EKF. It was emphasized that the discrepancy between the actual and prescribed noise covariance matrices seriously degraded the performance of the filters and induced divergence problems.

In order to resolve these problems, efforts have been devoted to the development of methodologies to estimate the noise parameters which characterize the noise covariance matrices. These methodologies can be classified into three categories [31]: covariance matching, correlation and Bayesian techniques. Covariance matching techniques [32] utilize the past data of the predicted state trajectories over either the entire duration or a moving time window to calculate the sample covariance matrix. Then, the noise parameters can be estimated from the sample covariance matrix and its residuals. However, biased estimation may be induced by the artificially designed time window. The performance of covariance matching techniques can be guaranteed only when the covariance matrix of the measurement noise is known [31,32]. Correlation techniques [33] attempt to estimate the noise parameters by using the auto-correlations between the residuals of the measurements and the corresponding estimated states. Based on this idea, the auto covariance least-squares method [34] was proposed to improve the performance of the traditional correlation techniques.

The third category is the Bayesian techniques, to which the proposed approach belongs. The Bayes' theorem is employed to formulate the posterior probability density function (PDF) of the noise parameters. By maximizing the posterior PDF, the optimal noise parameters can be obtained. Pioneering works [35–37] assumed a non-informative prior PDF so the maximization of the posterior PDF can be simplified as the maximization of the likelihood function. In [35], a moving horizon method was proposed to estimate the noise parameters for stationary situation. Selected horizons were imposed to formulate the measurement likelihood function. Although some criteria were given for the choice of the horizons, subjective judgment from the user was required for the determination of the trade-off level between the convergence and computational cost. In [36,37], the noise parameters were obtained by optimizing their posterior PDF with a half-or-double algorithm. The uncertainty of the estimated noise parameters can be quantified as an interpretation of the Bayesian inference. Again, this method can be applied only in an offline manner due to its substantial computational cost. On the other hand, the majority of the literatures [31–37] focused on stationary response only.

In this paper, a computationally efficient Bayesian approach is proposed for online estimation of the noise parameters. On the filter performance, it avoids the divergence problem and ensures the reliability of the uncertainty quantification. On the applicability, it allows for tracking the time-varying noise parameters so it can be applied to nonstationary situations. The proposed algorithm is systematic and computationally efficient for state estimation and it has promising potential for various engineering applications. These appealing features enhance the applicability and reliability of the EKF.

This paper is organized as follows. In Section 2, the EKF algorithm is briefly reviewed and the motivation of this study is revealed. In Section 3, the proposed Bayesian probabilistic approach is presented for online identification of the noise parameters. First, the parameterization of the noise covariance matrices is introduced. Then, the formulation of the proposed approach is presented. Thereafter, a computationally efficient online estimation scheme is presented. Finally, the procedure of the proposed algorithm is provided. In Section 4, two illustrative examples are presented to demonstrate the efficacy of the proposed approach. The first example refers to a 40-story shear building and the second example refers to a 5-degree-of-freedom Bouc–Wen hysteretic system.

2. Online updating of dynamical systems

2.1. Extended Kalman filter

Consider the state-space equation of general linear/nonlinear dynamical systems

$$\dot{\chi}(t) = \mathbf{G}(\chi, \mathbf{f}; \boldsymbol{\psi}) \tag{1}$$

where $\chi \in \mathbb{R}^{N_\chi}$ is the system state vector and $\boldsymbol{\psi} \in \mathbb{R}^{N_\psi}$ is the model parameter vector which characterizes the underlying dynamical system. The unmeasured external excitation (i.e., the process noise) $\mathbf{f}$ is modeled as an N_f-variate zero-mean Gaussian stochastic process.

Define the augmented state vector $\mathbf{y} \equiv [\chi^T, \boldsymbol{\psi}^T]^T \in \mathbb{R}^{N_y}$ that consists of the system state vector χ and the model parameter vector $\boldsymbol{\psi}$, where $N_y = N_\chi + N_\psi$. Then, the augmented state-space equation can be expressed as:

$$\dot{\mathbf{y}}(t) = \mathbf{g}(\mathbf{y}, \mathbf{f}; \boldsymbol{\psi}) \tag{2}$$

This equation can be linearized from an arbitrary state $\hat{\mathbf{y}}$ as follows:

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{B}\mathbf{f} + \tilde{\mathbf{g}} \tag{3}$$

where $\mathbf{A} = \partial\mathbf{g}/\partial\mathbf{y}|_{\mathbf{y}=\hat{\mathbf{y}}}$; $\mathbf{B} = \partial\mathbf{g}/\partial\mathbf{f}|_{\mathbf{y}=\hat{\mathbf{y}}}$; and $\tilde{\mathbf{g}} = \mathbf{g}(\hat{\mathbf{y}}, 0; \boldsymbol{\psi}) - \mathbf{A}\hat{\mathbf{y}}$.