Mechanical Systems and Signal Processing 66-67 (2016) 298–313

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Damage detection using frequency shift path

Longqi Wang, Seng Tjhen Lie, Yao Zhang *

School of Civil & Environmental Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798, Singapore

ARTICLE INFO

Article history:
Received 2 October 2014
Received in revised form
12 June 2015
Accepted 18 June 2015
Available online 16 July 2015

Keywords:
Auxiliary mass
Damage detection
Discrete time Fourier transform
Frequency shift path

ABSTRACT

This paper introduces a novel concept called FREquency Shift (FRESH) path to describe the dynamic behavior of structures with auxiliary mass. FRESH path combines the effects of frequency shifting and amplitude changing into one space curve, providing a tool for analyzing structure health status and properties. A damage index called FRESH curvature is then proposed to detect local stiffness reduction. FRESH curvature can be easily adapted for a particular problem since the sensitivity of the index can be adjusted by changing auxiliary mass or excitation power. An algorithm is proposed to adjust automatically the contribution from frequency and amplitude in the method. Because the extraction of FRESH path requires highly accurate frequency and amplitude estimators; therefore, a procedure based on discrete time Fourier transform is introduced to extract accurate frequency and amplitude with the time complexity of $O(n \log n)$, which is verified by simulation signals. Moreover, numerical examples with different damage sizes, severities and damping are presented to demonstrate the validity of the proposed damage index. In addition, applications of FRESH path on two steel beams with different damages are presented and the results show that the proposed method is valid and computational efficient.

1. Introduction

Structural safety and integrity have attracted many researchers' interest for the past decades because catastrophic failure is always associated with a great loss of human lives and wealth. Therefore, reliable techniques and equipment that are capable of detecting damages in structures in the early stage are very critical in avoiding such losses. Many damage detection methods, such as ultrasonic methods and radiography, are capable to detect and identify damages in practice. However, these methods usually need *a priori* information such as the vicinity of the damage. Fortunately, there are other alternative methods like vibration-based method which can locate the damage vicinity more effectively; and then this piece of information can be used for further damage assessment [1].

Typically, vibration-based methods use dynamic properties such as stiffness, mode shapes, natural frequencies or damping, to detect local damage in structures. Natural frequencies are easy to obtain and highly accurate [2]. A lot of works [3–10] have been carried out by using changes in frequencies in the past. Salawu [9] reviewed the damage detection methods using changes of natural frequencies and Fan [11] presented an update version of review later. Mode shapes are the orthogonal modes of the structures; they usually contain local information of structures and hence have been widely used for damage detection [1,11–17]. Frequency response function and operating deflection shape containing information of both natural frequencies and mode shapes are also frequently used in local damage detection [18–20]. Recently, the influence of auxiliary mass on a structure is studied [21,22]. Zhang et al. [23] proposed the idea of frequency shift curve by an auxiliary mass on a structure which can be used for

* Corresponding author.
E-mail address: zhangyao@ntu.edu.sg (Y. Zhang).

http://dx.doi.org/10.1016/j.ymssp.2015.06.028
0888-3270/

damage detection. Fang et al. [24] suggested that power of mode shape (PMS) could be extracted from Fourier transforms of acceleration signal and it could be used to detect damage in linear structures. Inspired by the ideas of frequency shift curve and PMS, a new concept named FREquency SHift (FRESH) path is introduced in this work for damage detection.

In the past, a lot of time-frequency algorithms have been developed and some of them have shown good performance in damage detection [25–27]. The most frequently used approach is Fast Fourier Transform (FFT). However, its frequency resolution is relatively low for extraction of FRESH path. Moreover, the error of estimated amplitude at natural frequency is relatively large as well [28,29]. Highly accurate frequency and amplitude are required in extracting the FRESH path, and therefore in this paper, the discrete time Fourier transform (DTFT) is used instead of FFT in estimating the natural frequencies and corresponding amplitude, due its higher accuracy for estimating the frequency and amplitude. By using DTFT to extract more accurate FRESH path and taking the information from frequency shifting and amplitude changing, the FRESH path produces better damage detection results.

The paper is organized as follows: Section 2 proposes the method to estimate natural frequencies and corresponding amplitude of a vibration signal by DTFT; Section 3 introduces the concept of FRESH path and its application in damage detection; Section 4 provides numerical verifications of the proposed method and discussion of influence of parameters; Section 5 shows experimental verification and finally, Section 6 presents main findings and conclusions.

2. Natural frequencies and amplitudes estimation using DTFT

In this section, the limitations of FFT are discussed and the method of extracting natural frequencies and corresponding amplitude by using DTFT is proposed, and then numerical studies are presented to verify the effectiveness and precision of DTFT.

2.1. Limitations of FFT

Because of good time efficiency, FFT is the most frequently used technology to transfer time domain data into frequency domain. However, the frequency resolution of FFT is relatively low, it may introduce a maximum frequency estimate error of $\pm 0.5\Delta f$, where Δf is the frequency resolution of FFT. Frequency resolution of FFT depends on the number of points in the transform. High frequency resolution needs more transform data which increases the computational time and memory cost. Another major limitation of FFT is the leakage effect making the estimation error of amplitude even worse. A useful strategy to overcome this is to add a window onto the signal. However, if a highly accurate frequency or amplitude is required, this strategy still cannot solve the problem completely. In this section, the basic formula of Fourier transform is revisited, and it is shown that it is possible to develop a procedure to extract frequencies of a signal with high degree of accuracy but with the same time complexity.

2.2. Theory of using DTFT as natural frequencies and amplitudes estimator

2.2.1. Relationship of CTFT and DTFT of a signal

The very first Fourier Transform is the Continuous Time Fourier Transform (CTFT) defined as

$$X(f)=\int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}\,\mathrm{d}t \tag{1}$$

where $x(t)$ is the input signal.

Usually, CTFT converts a continuous time domain data $x(t)$ into a continuous frequency domain data $X(f)$, and there is an inverse transform to reconstruct frequency domain data back into time domain. When the input signal of CTFT is a discrete sequence, integral in CTFT becomes summation; it is called DTFT and given by

$$X_D(\widehat{f})=\sum_{n=-\infty}^{+\infty} x[n]e^{-j2\pi n\widehat{f}} \tag{2}$$

in which $x[n]$ is discrete time sequence and $\widehat{f}$ is a unit-less frequency variable.

It is shown clearly in Eq. (2) that DTFT converts a discrete sequence into a continuous curve in frequency domain. Then, it is interesting to know how the continuous curve $X_D(\widehat{f})$ is related to the frequency representation $X(f)$ by CTFT, if $x[n]$ is a sequence sampled from a continuous time signal $x(t)$ at an equal interval of T.

The sampled time domain signal $s(t)$ can be written as

$$s(t)=\sum_{n=-\infty}^{+\infty} x[n]\delta(t-nT)=x(t)\sum_{n=-\infty}^{+\infty}\delta(t-nT) \tag{3}$$

By using the convolution rule, CTFT of $s(t)$ can be obtained as

$$S(f)=X(f)*\mathscr{F}\left(\sum_{n=-\infty}^{+\infty}\delta(t-nT)\right) \tag{4}$$

in which $\mathscr{F}$ is CTFT of a signal.