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The instantaneous frequency rate spectrogram

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ABSTRACT

An accelerogram of the instantaneous phase of signal components referred to as an instantaneous frequency rate spectrogram (IFRS) is presented as a joint time-frequency distribution. The distribution is directly obtained by processing the short-time Fourier transform (STFT) locally. A novel approach to amplitude demodulation based upon the reassignment method is introduced as a useful by-product. Additionally, an estimator of energy density versus the instantaneous frequency rate (IFR) is proposed and referred to as the IFR profile. The energy density is estimated based upon both the classical energy spectrogram and the IFRS smoothened by the median filter. Moreover, the impact of an analyzing window width, additive white Gaussian noise and observation time is tested. Finally, the introduced method is used for the analysis of the acoustic emission of an automotive engine. The recording of the engine of a Lamborghini Gallardo is analyzed as an example.

1. Introduction

The classical spectrogram is the graphical representation of a signal. It is the estimate of a signal's energy distributed over the time–frequency (TF) plane, therefore it can be referred to as an energy spectrogram. This distribution is estimated based upon the squared magnitude of the STFT of the signal. Moreover, different kinds of spectrograms are also used to visually present other features of signals, for example, the STFT phase or instantaneous frequency. In [19] the author respectively introduced a phase spectrogram and a frequency spectrogram. Following these examples in this paper a new kind of spectrogram is introduced known as "an instantaneous frequency rate spectrogram". This expresses the instantaneous frequency rate distribution in the time–frequency domain. The new distribution together with the classical energy spectrogram can be useful in order to analyze mechanical vibrations or acoustical signals as well as all other signals, including multicomponent and nonstationary signals, for which the STFT can be calculated and which are signals that have a limited frequency band.

On the other hand, the IFR spectrogram can be interpreted as a phase accelerogram. Although an accelerogram in mechanics is commonly known as a graphical presentation of the kinematic acceleration of an observed object, usually it is a plot, where the acceleration is plotted as a function of time or location. However, in this paper, the phase accelerogram is presented as a synonym of the IFRS. The representation approximates the instantaneous phase acceleration of the amplitude modulated and frequency modulated (AM · FM) components of a signal. The estimated acceleration values are localized on the TF plane according to the classical energy spectrogram of the analyzed signal. The phase acceleration value is also

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commonly known as the sweep rate of a chirp or the chirp rate. The subject of the time–frequency (or time-scale) analysis is still ongoing whose contributions confirm, for example [8,20,25], also in signal processing in mechanics [6,15,27,28,30]. The TF analysis is dedicated especially to the analysis of nonstationary signals that are frequent in mechanical diagnosis or identification [1,9,26,31].

Let us consider a multicomponent complex signal consisting of N mono-components that are described in the following manner:

$$u(t) = \sum_{n=1}^{N} u_n(t) = \sum_{n=1}^{N} a_n(t)\exp(j\varphi_n(t)), \quad (1)$$

where $u_n(t)$, $a_n(t) > 0$, and $\varphi_n(t)$ denote, respectively, the waveform, the envelope, and the instantaneous phase of the n-th mono-component. Here, j is an imaginary unit, $j^2 = -1$ and exp() represents an exponential functor. Thus the instantaneous frequency rate as a function of time corresponding to the n-th component is defined as

$$R_n(t) = \frac{1}{2\pi} d^2\varphi_n(t)\, \mathrm{d}t^2 = \mathrm{d}F_n(t)/\mathrm{d}t, \quad (2)$$

where $F_n(t)$ is the instantaneous frequency of the component expressed in Hertz. In other words, the proposed phase accelerogram is interpreted as a common distribution determined by $R_n(t)$ curves for all values of n. The term “component” refers to a waveform that is unambiguously described by both $a_n(t)$ and $\varphi_n(t)$. The conditions that must be fulfilled by these curves are defined in [10,13,29], amongst others.

The instantaneous frequencies of components can be relatively simply calculated by the differentiation of the STFT phase with respect to time. Unfortunately, the IFR cannot be obtained by similar second order differentiation. Therefore Nelson proposes the use of the second order mixed derivative of the STFT phase in order to distinguish stationary and pulse components [22]. Here, this approach is extended and the obtained transform has a strictly physical interpretation – that is to say, phase acceleration.

Theoretically, the IFR of a waveform, which stands out clearly from the noise, can be estimated by computing its slope within the TF plane. To a certain extent, the method proposed in this paper relates to this approach. However, the IFR can be obtained not only from the top of the ridges (vertex) of the energy spectrogram, but also at each point of the TF plane. Therefore, we do not need to look for vertices. Each portion of the distributed energy can be associated with the estimated IFR. Moreover, this novel method allows us to avoid any inconvenience that occurs during the analysis of signals characterized by the amplitude modulation, through the use of a novel demodulation method.

Some conceptions of IFR estimation based upon the TF analysis are presented in the following selected publications: [4,23,24,32]. This paper is one more contribution to this subject.

The advantage of the proposed method is the obtainment of IFR values without the comparison of many transforms. Meanwhile, the author of [32] presents a method based upon the discrete chirp-Fourier transform that is, in fact, a set of (quadratic) chirplet transforms [21], obtained using windows characterized by different instantaneous frequency rates. In contrast, the method presented in this paper requires the calculation of only one STFT for one selected analyzing window. It seems that such an approach leads to a reduction in numerical cost, amongst others. In addition to this, the method introduced here is designed for the analysis of any mono- and multicomponent AM · FM signals whose components are sparsely distributed on the TF plane.

The author of [4] proposes the use of a combined Wigner–Hough transform in order to estimate the IFR. Firstly, he proposes the transformation of a signal into a TF representation, referred to as the Wigner distribution. Subsequently, the representation is transformed once again into the Hough transform which is a particular variant of the Radon transform, well known in image processing. However, in the method proposed by Barbarossa [4], the information of the IFR cannot simply be assigned to the TF plane.

In [24] the authors present a method for finding the sparse decomposition of a signal. This method leads us to obtaining a set of components that are characterized by the IFR, amongst others. The method is iterative and the maximum likelihood estimation is applied. Conceptually, the approach is quite different in comparison with the presented method and, as the authors point out, its computational complexity is relatively high.

The relationships between the phase and the magnitude of STFT are the subject of [2], but the authors have not introduced direct relationships between STFT and the instantaneous frequency rate of signal components. Therefore, this contribution can only be a supplement to these considerations.

This paper is organized as follows. The estimator of an IFR dedicated only to FM signal analysis is defined in Section 2. Following this a universal estimator dedicated to the analysis of any AM · FM signals, a novel method of demodulation in the TF domain, and the introduction of an additional median filtering in order to smoothen the distributions are presented in the same section. Section 3 contains a description of the implementation of the introduced estimator. An IFR profile is estimated based upon the statistics of obtained accelerograms and classical energy spectrograms in Section 4. In this section a consideration of both the impact of AWGN and the analyzing window width on the estimation is also put forward. In Section 5 an example of the usage of the method in the analysis of the acoustic emission of a mechanical engine is shown. Finally, Appendix contains analytical considerations which use the IFR estimator in order to analyze any LFM signal.