Mechanical Systems and Signal Processing 66-67 (2016) 546–556

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Inverse Gaussian mixtures models of bearing vibrations under local faults

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ARTICLE INFO

Article history:
Received 25 April 2014
Received in revised form
20 February 2015
Accepted 9 May 2015
Available online 4 June 2015

Keywords:
Localised surface bearing faults
Inverse Gaussian distribution
Fault modelling
Bayes' factor
Variable rotational speed

ABSTRACT

Repetitive impacts performed by damaged spot on a component of the rolling element bearing specific statistical properties, due to the constant angular distance between the roller elements. Under (almost) constant rotational speed the successive impacts are regarded as almost periodic with small random fluctuations due to slippage. Often these random components are modelled as normally distributed, which is unrealistic since physically impossible events, such as negative time between two consecutive impacts, become likely by the nature of the distribution. Motivated by this deficiency we propose a new model that describes the occurrence of repetitive vibrational patterns as realisation of a point process with the (mixture) inverse Gaussian distribution of the inter-event times. Such a model is applicable to both constant and variable rotational speeds. Additionally, the proposed model inherently describes the quasi-cyclostationarity of the impact times under almost constant rotational speed. The applicability of the model was evaluated using vibrational signals generated by bearings with localised surface fault.

1. Introduction

Bearing faults among the most common causes for mechanical failures [1,2]. Commonly, methods based on analysis of vibrational signals focus on extracting specific frequency components that are linked to particular surface faults [3]. Inferring about bearing condition using such a feature set is possible if the monitored bearing operates under constant and known rotational speed. However, in reality rotational speed fluctuates so that the effectiveness of these features is significantly reduced. In this paper we model the vibrational patterns, generated by bearings with localised surface fault, as a point process with inverse Gaussian mixture inter-event distribution.

The main source of information for detecting localised bearing faults is the time occurrences of particular vibrational patterns. The observation of these repetitive vibrational patterns through the concepts of point processes was firstly proposed by Antoni and Randall [4]. The proposed model was based on periodic impact occurrences with small variations in the period modelled as Gaussian random variable. In a similar approach Borghesani et al. [5] analysed the distribution of the times between consecutive impacts that emerge under non-stationary but known operating conditions by allowing significant speed variations and by calculating the squared envelope spectrum coupled with computed order tracking. The approach proposed in this paper goes one step further by removing the limitation of constant and known operating conditions.

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http://dx.doi.org/10.1016/j.ymssp.2015.05.010

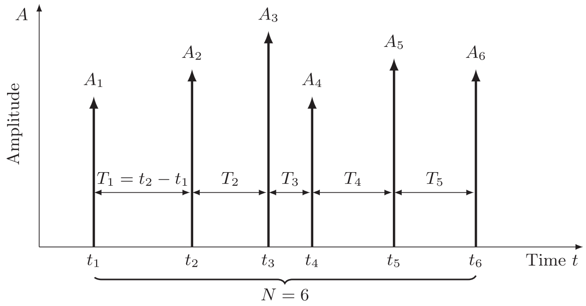

Fig. 1. Realisation of a point process.

Despite the effectiveness of the proposed models, they suffer from two major deficiencies. Firstly, the time intervals between two consecutive impacts are modelled as Gaussian random variables. Since the support of the Gaussian distribution is on the interval $(-\infty, \infty)$, it is likely that a time increment takes negative value which is physically impossible. The second deficiency regards the capability of modelling significant speed fluctuations. By incorporating random speed fluctuations one significantly increases the complexity of currently adopted models. Therefore, we propose a novel model that describes the impact occurrences, generated by localised bearing surface damage, as realisation of a point process whose inter-event times are governed by (mixture) inverse Gaussian (IG) distribution. By using such an approach one can construct a unified model capable of describing both single and multiple bearing faults regardless of the speed fluctuations. We have to note that in this paper we do not deal with fault diagnosis itself. However, the diagnostic approach proposed in [6] for a particular instance of the problem treated below is readily applicable for the results derived in this paper.

The paper is organised as follows. The idea of point processes, which are basic building blocks of the model, are presented in Section 2. The definition and essential statistical properties of the inverse Gaussian distribution and its mixture variant are presented in Section 3. Applicability of the proposed framework for modelling localised bearing faults is presented in Section 4. The question of whether to use pure or mixture IG distribution based on Bayes' factor is analysed in Section 5. Finally, the simulated and experimental results are presented in Section 6

2. Basics of point processes

Point processes represent a branch of the theory of random processes that are most commonly used for characterising random collections of point occurrences [7]. In the simplest form, these points usually represent the time instances of their occurrences $\dots, t_{i-1}, t_i, t_{i+1}, \dots$ with the corresponding amplitudes $\dots, A_{i-1}, A_i, A_{i+1}, \dots$, as shown in Fig. 1. A point process can be specified in several equivalent ways [8]:

- through the non-negative number $N \in \mathbb{Z}^+$ that specifies the number of observed occurrences within some time interval;
- through the distribution of the inter-event times $\{T_1, \dots, T_n\}$ where $T_i = t_i - t_{i-1}$; and
- through the frequency with which events occur around the time instance t with respect to the history $\mathcal{H}_t$ of the process; this statistical property is usually called conditional intensity function $\lambda(t|\mathcal{H}_t)$.

The amplitudes A_i, on the other hand, can be regarded as independent of event times t_i and are not of interest in this paper.

The simplest point process is the Poisson process for which the probability of observing k events in a time interval $[0, t]$ follows the Poisson distribution:

$$P(N=k) = \frac{e^{-\lambda t}(\lambda t)^k}{k!}, \tag{1}$$

where the parameter λ is referred to as the intensity of the process. Equivalently, the distribution of the inter-arrival times T_i follows the exponential distribution:

$$p(t) = \lambda e^{-\lambda t}. \tag{2}$$

Finally, for the Poisson process, the intensity function $\lambda(t|\mathcal{H}_t)$ is constant $\lambda(t|\mathcal{H}_t) = \lambda$ and is independent of the history of previous events. In the remaining of the paper, we will use abbreviated notation for probability distribution function (PDF) $p(x)$ instead of $p_X(X=x)$, meaning the PDF of the random variable X takes value x.

The concept of point processes can be applied to describe the impact time occurrences typical for bearings with localised surface faults. In what follows, a point process whose inter-arrival times follow IG distribution is analysed for the purpose of describing localised surface bearing faults.