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# Partial pole assignment with time delay by the receptance method using multi-input control from measurement output feedback

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ABSTRACT

This study investigates the receptance method for the partial pole assignment of time-delay nonlinear systems using multi-input control from measurement output feedback (i.e., acceleration, velocity and displacement). The receptance method has a remarkable advantage compared to other methods in that there is no need to know the mass, damping and stiffness matrices of the system, which are typically obtained from the finite element method. We achieve partial assignment of the desired poles with no spillover using the assigned and unchanged poles and the corresponding eigenvectors of the closed-loop system. We used different types of generalised inverse matrices to obtain the realisable control gains. The modal constraints for the assigned eigenvectors are thus obtained. Because certain components of the measurement output were found to be unmeasurable, a numbering system is proposed for determining zero elements in the control gains. Then, realisable control gains are obtained after zero-column substitutions are made in the corresponding matrix with the numbering system. Our theoretical results show that having multi-input control from the measurement output feedback is effective for a partial pole assignment with time delay in structures. This theory is demonstrated by several numerical examples of a three-degree-of-freedom damped mass-spring system.


## 1. Introduction

Over the last few decades, the active control of vibrational systems is a topic that has been widely studied [1,2]. It is well recognised that the convergence of a linear vibratory system is determined by its poles. It was shown that the poles of a controllable system can be assigned by state feedback [3]. Conventional pole assignment problems are solved using finite element (FE) models or theoretical models. However, FE models have several disadvantages, e.g., a difficulty in obtaining precise damping models of real structures, especially for structures with many forms of damping (such as friction and viscosity) [4]. Theoretical models unavoidably involve errors with respect to practical systems. To overcome these shortcomings, the receptance method for linear active vibration control has recently been developed by Mottershead et al. [4–6]. Poles and/or zeros can be assigned as required using the receptance method with state feedback. One remarkable advantage of this method is that it is entirely based on

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## Nomenclature

*Symbols*

| | |
|---|---|
| $\mathbf{B}$ | force selection matrix |
| $\mathbf{b}_j$ | $j$th column of $\mathbf{B}$ |
| $\mathbf{C}$ | damping matrix |
| � | set of complex numbers |
| $c_1, c_2, c_3$ | damping coefficients |
| $\mathbf{D}$ | matrix consists of $\mathbf{D}_a$, $\mathbf{D}_v$, $\mathbf{D}_d$ |
| $\mathbf{D}_a$, $\mathbf{D}_v$, $\mathbf{D}_d$ | measurement choosing matrices |
| $E_{wv}$ | set of closed-loop eigenvectors |
| $E'_{wv}$ | set of closed-loop eigenvectors of the system without a time delay when applied control gains with time delay |
| $\mathbf{e}_i, \mathbf{e}_j$ | $i$th, $j$th unit vector, respectively |
| $\mathbf{F}(t)$ | control force vector |
| $F_1, F_2, F_3$ | control forces |
| $\mathbf{G}_a$, $\mathbf{G}_v$, $\mathbf{G}_d$ | control gains matrices |
| $\mathbf{H}(*)$ | receptance matrix of the open-loop system when the complex frequency is equal to $*$ |
| i | imaginary unit |
| $i, i_1, i_2$ | integer variables |
| $j$ | integer variable |
| $\mathbf{K}$ | stiffness matrix |
| $k, k_u$ | integer variables |
| $k_1, k_2, k_3$ | stiffness coefficients |
| $\mathbf{M}$ | mass matrix |
| $m$ | number of control inputs |
| $m_1, m_2, m_3$ | masses |
| $N_0$ | number set whose elements are the zero-column numbers |
| $n$ | number of degrees of freedom of the system |
| $P_c$ | set of closed-loop poles |
| $P_c'$ | set of closed-loop poles of the system without a time delay when applied control gains with time delay |
| $P_o$ | set of open-loop poles |
| $p$ | number of poles to be assigned |
| $\mathbb{R}$ | set of real numbers |
| $\mathrm{Re}(*)$ | the real part of $*$ |
| $r$ | integer variable |
| $s$ | complex Laplace frequency |
| $t$ | time |
| $\mathbf{u}(t)$ | multi-input control |
| $\mathbf{V}$ | matrix associated with the unchanged poles |
| $\mathbf{V}_k$ | matrix associated with the $k$th unchanged pole |
| $\mathbf{v}_k$ | $k$th unchanged eigenvector |
| $\mathbf{W}$ | matrix associated with the assigned poles |
| $\mathbf{W}_k$ | matrix associated with the $k$th assigned pole |
| $\mathbf{w}_k$ | $k$th assigned eigenvector |
| $w_{k,i}, w_{k,j}$ | $i$th, $j$th element of $\mathbf{w}_k$, respectively |
| $\mathbf{x}(t), \dot{\mathbf{x}}(t), \ddot{\mathbf{x}}(t)$ | displacement, velocity, acceleration vector, respectively |
| $x_1, x_2, x_3$ | generalised coordinates |
| $\boldsymbol{\alpha}$ | vector reshaped from $\mathbf{G}_a\mathbf{D}_a$, $\mathbf{G}_v\mathbf{D}_v$, and $\mathbf{G}_d\mathbf{D}_d$ |
| $\boldsymbol{\alpha}_{a,j}, \boldsymbol{\alpha}_{v,j}, \boldsymbol{\alpha}_{d,j}$ | $j$th row of $\mathbf{G}_a\mathbf{D}_a$, $\mathbf{G}_v\mathbf{D}_v$, $\mathbf{G}_d\mathbf{D}_d$, respectively |
| $\boldsymbol{\beta}$ | constraint vector of the assigned poles |
| $\boldsymbol{\beta}_k$ | constraint vector associated with the $k$th assigned pole |
| $\beta_{\mu_k,j}$ | constraint associated with the $k$th assigned pole |
| $\kappa_{k,ij}$ | modal ratio constraint of the $k$th assigned eigenvector |
| $\boldsymbol{\Lambda}$ | matrix associated with the closed-loop poles |
| $\boldsymbol{\Lambda}\{1\}$ | set of all $\{1\}$ generalised inverse matrices of $\boldsymbol{\Lambda}$ |
| $\boldsymbol{\Lambda}^{(1)}$ | one specific $\{1\}$ generalised inverse matrix of $\boldsymbol{\Lambda}$ |
| $\boldsymbol{\Lambda}\{1,3\}$ | set of all $\{1, 3\}$ generalised inverse matrices of $\boldsymbol{\Lambda}$ |
| $\boldsymbol{\Lambda}^{(1,3)}$ | one specific $\{1, 3\}$ generalised inverse matrix of $\boldsymbol{\Lambda}$ |
| $\lambda_k$ | $k$th unchanged pole |
| $\mu_k$ | $k$th assigned pole |
| $\tau$ | time delay |
| $\boldsymbol{\Psi}_{\mu_k}$ | constraint matrix associated with the $k$th assigned pole |
| $\boldsymbol{\Psi}_{\mu_k,j}$ | constraint vector associated with the $k$th assigned pole |

*Superscripts*

| | |
|---|---|
| T | transpose |
| – | conjugation |
| · | first-order derivative with respect to $t$ |
| ·· | second-order derivative with respect to $t$ |

*Special functions*

| | |
|---|---|
| $N(*)$ | function of numbering a specific component $*$ |

data from modal testing [4] with no need to know the exact mass, damping and stiffness matrices, which are typically obtained using the FE method.

The receptance method has been further developed by Mottershead and his colleagues since its first introduction into linear systems theory. Besides state feedback, output feedback and measurement output feedback have also been used for vibration control. Measurement output feedback features the feedback consisting of any kinetic measurements, while output feedback uses only the output which is actually a combination of the state variables. Mottershead et al. [7] used the receptance method to achieve vibration absorption and detuning in structures to avoid resonance through the use of output feedback instead of state feedback. Output feedback control has the advantage of enabling the collocation of actuators and sensors. Singh et al. [8] developed a linear control strategy by multiple-input from measurement output feedback based on the receptance method. The results of this study showed that measurement output feedback was more flexible and applicable than state feedback. Partial pole assignment without spillover is necessary when only part of the open-loop poles need to be assigned. Tehrani et al. [9] then assigned partial poles to