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Identification of two cracks in a rod by minimal resonant and antiresonant frequency data

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ABSTRACT

In this paper we consider the identification of two cracks of equal severity in a uniform free–free rod under longitudinal vibration. Each crack is simulated by a translational spring connecting the two adjacent segments of the rod and the cracks are considered to be small. We show that the inverse problem can be formulated and solved in terms of three frequency data only, corresponding to a suitable set of low resonant and antiresonant frequencies. Closed-form expressions of the damage parameters in terms of the measured frequency shifts are obtained. The paper improves existing results available in the literature, since the use of antiresonant frequencies allows us to exclude all the symmetrical crack locations occurring when only natural frequency are used as data. The analysis also explains why the use of high frequency data introduces spurious damage locations in the inverse problem solution. Numerical simulations show that if accurate input data are available then damage identification leads to satisfactory results.

1. Introduction

Dynamic methods are a useful diagnostic tool for several applications in mechanical and civil engineering. Diagnostic techniques based on natural frequency data, in particular, have the advantage of being simple to carry out in practice, since they require a limited experimental commitment and can be easily repeatable during the structure's lifetime. In addition, frequencies can be measured more easily than can mode shapes, and are less affected by experimental errors. This class of diagnostic methods operates on a global scale and does not require a priori information on the damaged area. Their global character, however, has the disadvantage of introducing synthetic information on the formulation of the inverse problem. Therefore, to be effective, these techniques often need additional information, such as a knowledge of the undamaged configuration and of the characteristics of the defect to be identified (localized or diffuse damage, for example). Another aspect worth of noticing is the relatively low sensitivity of the natural frequencies to damage, see, among other contributions, [1,2]. This is a peculiarity also of other damage indicators [3], and negative effects can be controlled by reducing the experimental errors and using accurate mechanical models for the interpretation of the measurements [4].

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As a confirmation of the growing interest on this class of diagnostic methods, we recall that an extensive line of research on damage identification by frequency data has been developed over the past three decades since the pioneering paper by Adams et al. [5]. The damages commonly considered have localized nature, are notches or cracks, as this class of defects frequently occurs in engineering applications. Most of the results concern damage identification in one-dimensional elements, such as beams or bars. Indeed, in addition to the high number of applications, cracked beams have the advantage of being described by consolidated mechanical models and being relatively manageable from the mathematical point of view, at least in the case of cracks that remain open during the vibration of the system. Of course, there are contexts in which it is necessary to take into account the phenomena of opening/closing of cracks, and more sophisticated nonlinear models of crack must be implemented, see, among other contributions, [6] and [7] for an analysis of the direct problem in beams and rotors, respectively, and [8] for the identification of breathing cracks in a vibrating beam. Without claiming of completeness, the reader may refer to [9–13] for an overview of some recent contributions on damage detection based on frequency data.

Despite the extensive literature on identification of cracks in rods and beams by frequency data, some basic problems are still open. Among the problems for which a satisfactory knowledge is not available yet, there is the identification of multiple cracks.

One of the first contributions to the treatment of the direct problem is due to Ostachowicz and Krawczuk [14], who considered the effect of two open cracks on the lower natural frequencies and vibration modes of a cantilever beam in bending. Each crack was modelled as a massless linear elastic rotational spring located at the cracked cross-section, according to arguments of Fracture Mechanics [15]. Later on, Ruotolo and Shifrin [16] presented an efficient technique for solving the eigenvalue problem of the free bending vibration of a multicracked beam. Main advantage of their approach was in the reduction of the differential equations between cracks to a single differential equation on the whole beam axis interval. The method was later applied to longitudinal vibration of a multi-cracked rod by Ruotolo and Surace [17]. In this case, each crack is included in the one-dimensional rod model as a massless linear elastic translational spring located at the damaged cross-section. Among the recent contributions, worth of mention is the study developed by Caddemi and Caliò [18], who derived exact closed-form solutions for the free-vibration of a uniform Euler–Bernoulli beam in the presence of multiple open cracks mathematically modelled as Dirac's delta functions in the bending stiffness coefficient. Other interesting contributions to the direct problem appeared in the last few years, but, since our main goal is the analysis of the inverse problem, we refer the reader to the introduction of the paper [18] for an updated overview on this topic.

Results on the inverse problem of identifying multiple (open) cracks in rods and beams from frequency data are less numerous, see the updated state-of-the-art on identification and conditioning monitoring for multi-cracked structures by Sekhar [19] (Section 5), and [20] for an application to model-based identification of two cracks in a rotor system.

Assuming as above the linear concentrated flexibility model to describe cracks in rods and beams, one approach consists in considering as many natural frequencies as the unknowns of the problem (two unknowns for each crack, the position and the severity), and then solve the system formed by the characteristic equation written for all the natural frequencies in terms of the damage parameters, see, for example, [21,22]. Inverse transcendental eigenvalue problems for the identification of multiple open cracks in a longitudinally vibrating rod were considered by Singh [23]. An iterative procedure based on a suitable Taylor series expansion of the system of characteristic equations for the damaged rod was used to estimate the damage parameters. Singh noticed that the possible presence of spurious solutions can be avoided by carefully selecting the data and using simultaneously natural frequency and antiresonant frequency measurements. This is a powerful class of methods, but has the drawback of requiring a strong support on numerical simulation, with the consequence of making difficult to find out general properties, such as, for example, the indication of optimal data to be used in order to reduce the non-uniqueness effects in the inverse problem solution.

Another approach to multi-cracked identification transforms the inverse problem to an optimization issue. It consists in determining the damage parameters such that the natural frequencies of the mechanical model are closest (in some least square sense) to those found experimentally, see [24,25] and, for a linearized version suitable in the case of small cracks, [26,12]. An error function which measures the distance between experimental and analytical frequency values is minimized via gradient-type methods. This class of techniques allows us to dealing with a large number of cracks and system of high complexity (beams of variable profile under general set of end conditions), but the approach has several drawbacks mainly connected with the non-convexity of the error function and, as a consequence, with the appearance of several local and global minima. Basic questions such as how accurate the description of the reference configuration has to be or how many data are necessary to ensure uniqueness of the solution (at least in local sense) are rarely discussed in the literature and are mainly still open.

In the case of multi-cracked rods under longitudinal vibration, an attempt to use the classical results of the spectral theory for Sturm–Liouville operators has been done in [27]. The authors proved that knowledge of the highest part of a single spectrum of a rod with multiple cracks suffices to determine uniquely the (unordered) set of lengths of the segments of bar separating the cracks. Unfortunately, no reconstruction algorithm for the position of the cracks was provided in that study. The most general result along this direction has been obtained by Shifrin [28]. Shifrin proposed a constructive procedure for identifying the position and severity of multiple cracks in a longitudinally vibrating rod by the knowledge of two full spectra corresponding to different boundary conditions. It is worth noticing that, different from most of the methods available in the literature, Shifrin's technique does not assume that the number of cracks is known. However, the reconstruction procedure needs a large number of eigenvalues (independently on the number of cracks) to obtain stable and accurate estimates: two infinite spectra, in principle, or at least a number of eigenvalues of the order of 10 for each spectrum in the applications presented in the above-mentioned paper [28]. This seems to be a possible limitation of the method because, first, it is difficult to obtain accurate measurement of high frequencies in practice and, second, because the analytical model of the rod under longitudinal vibration loses accuracy as the eigenvalue order increases.