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Influence of indexing errors on dynamic response of spur gear pairs

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ABSTRACT

In this study, a dynamic model of a spur gear pair is employed to investigate the influence of gear tooth indexing errors on the dynamic response. This transverse-torsional dynamic model includes periodically-time varying gear mesh stiffness and nonlinearities caused by tooth separations in resonance regions. With quasi-static transmission error time traces as the primary excitation, the model predicts frequency-domain dynamic mesh force and dynamic transmission error spectra. These long-period quasi-static transmission error time traces are measured using unity-ratio spur gear pairs having certain intentional indexing errors. A special test setup with dedicated instrumentation for the measurement of quasi-static transmission error is employed to perform a number of experiments with gears having deterministic spacing errors at one or two teeth of the test gear only and random spacing errors where all of the test gear teeth have a random distribution of errors as in a typical production gear.

1. Introduction

Every manufactured gear contains certain types and magnitudes of errors depending on the quality level imposed. Such errors often contribute to the loaded motion transmission error to affect the meshing dynamics of gears. Consequently, understanding the impact of different gear design and manufacturing based errors and tolerances on the dynamic transmission error of gears is crucial. One of the most significant contributors to the gear transmission error is the tooth indexing errors. Gear tooth indexing error (deviation) is defined as the displacement of any tooth flank from its theoretical position relative to a reference tooth flank [1]. In relation to it, tooth spacing error is defined as the circumferential position error of one gear tooth flank with respect to the previous tooth flank. Ideally, a particular gear with Z number of teeth has identical involute profiles equally spaced around the pitch diameter. Existence of indexing error means that some of the tooth profiles are angularly misplaced from their ideal position with respect to one randomly chosen reference profile (index tooth or profile), say Tooth-1 without loss of generality (Fig. 1). The right hand side flank of Tooth-1 is the reference profile (flank) when certain amount of torque acting in the clockwise direction is assumed to exist on this gear. The circular distances, S1 and S2, between the right flanks of Tooth-1 and Tooth-2 and also between Tooth-2 and Tooth-3, where both flanks intersect the reference diameter are both equal to a circular pitch p ($p=\pi m$, where m is the module) for a gear with ideal geometry. If $S1$ deviates from the nominal circular pitch p ($S1 \neq p$), then the difference is

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interpreted as the spacing error ε_1 for Tooth-2. Similarly, if $S2$ has a different values than p then it is interpreted as the spacing error ε_2 for Tooth-3. The value $\varepsilon_1+\varepsilon_2$ becomes the indexing error for Tooth-3. If spacing error of any Tooth-N of a gear is ε_{N-1}, then the corresponding indexing error for Tooth-N is $\sum_{j=1}^{N-1} \varepsilon_i$, where j is the indexing error index.

Gear tooth indexing errors arise during manufacturing, causing deviations related to the cutting or heat treatment process in addition to the random components [2]. Indexing errors modify the transmission error as they cause a certain gear tooth profile to be misplaced on the reference diameter, thus either coming into contact earlier or later with the corresponding tooth on the mating gear compared to its expected nominal timing under ideal conditions. This essentially shifts the contact in time that can significantly change the dynamic behavior of the gears as the dynamic excitation phase continuously changes and instantaneous contact ratio becomes lower or higher than expected at different times causing either overloading or contact loss of the tooth in mesh. Consequently, complicated indexing error patterns that interact with each other on gears in mesh could significantly alter the resultant life of gears under operation. The frequency spectra for the dynamically mistuned (deviations from the originally symmetric geometrical attributes which affect the gear dynamic response) gears with indexing errors show significant increase to the non-harmonic gear mesh orders, making the spectra broad-band [3]. One of the main reasons for these non-harmonic orders to exist is the fluctuations in the transmission error values due to spacing/indexing errors. Therefore, it is not sufficient to use limited Fourier series amplitudes of transmission error to simulate the mistuned meshing dynamics of gears any more. The proper means of simulating indexing errors would be to apply the errors over multiple revolutions of both pinion and gear, covering their total hunting period. Worst case spacing errors occur when the respective errors of the pinion and gear match up.

The published work on the effects of indexing errors on gear dynamics is rather sparse. Remmers [2] developed an analytical method to study the effect of tooth spacing errors, load, contact ratio and profile modifications on the gear mesh excitations. He indicated that random tooth spacing errors may be used to reduce the gear mesh excitations at certain frequencies. Mark [3,4] derived expressions for Fourier Series coefficients of all components of static transmission error including harmonic and non-harmonic coefficients of gear defects of concern. He used two-dimensional Fourier transforms of local tooth pair stiffness and tooth surface deviations from perfect involute to come up with these expressions and used them to study mesh transfer functions of gears with different surface and profile deviations. Kohler and Regan and later Mark [5–7] discussed components of the frequency spectrum for gears with pitch errors based on analytical approaches and agreed on the fact that existence of the components depends on loading conditions and if the only deviation from perfect tooth geometry is due to pitch errors then frequency spectrum of corresponding transmission error function will have no components at the mesh frequency harmonics. Padmasolala et al. [8] developed a model to understand the effectiveness of profile modification for reducing dynamic loads in gears with different tooth spacing errors. They showed that linear tip relief is more effective in reducing dynamic loads on gears with relatively small spacing errors whereas parabolic tip relief becomes more effective when the amplitude of the spacing error increases. Wijaya [9] studied effects of spacing errors and run-out on the dynamic load factor and the dynamic root stress factor of an idler gear system. He employed an analytical approach that defines the static transmission error and static tooth forces and predicted dynamic mesh force spectra of an idler gear system using a linear, time-invariant model. C. Spitas and V. Spitas [10] also investigated overloading of gears and effect of tip relief on the dynamics of gears with indexing errors. They employed a geometry-based meshing analysis with a multi degree-of-freedom dynamic model and reported simulated load factors and transmission error functions for gears with assumed indexing errors. Milliren [11], Handschuh [12] and Handschuh et al. [13] investigated the influence of various gear errors on the quasi-static transmission errors and root stresses of spur gears experimentally. They used the same test rig to investigate the effects such as spacing errors and lead wobble on the transmission error. Moreover, they compared experimental results with the results of a contact model and showed that the results are highly correlated. Recently, Bihr et al. [14] developed a dynamic model to predict the noise radiation from automotive transmissions. This model is capable of evaluating noise radiation from gear pairs with different transmission error specifications.

In this study, influence of indexing errors on dynamics of spur gear pairs is investigated. An experimental test setup and its encoder based measurement system are used to measure long-period loaded transmission error excitations. A dynamic model capable of including long-period transmission error excitations is proposed to demonstrate the effect of indexing errors on the resultant mistuned dynamics of gears. The model presented in this study is unique in the way it includes measured long-period, broad-band quasi-static transmission error (STE), regardless of its resolution, from any parallel axis gear set with any design, loading condition and speed in order quantify the resultant dynamic gear mesh force, dynamic transmission error (DTE), dynamic bearing forces and gear deflections. The main focus of this study is on the off-resonant dynamics of spur gears. Resonant conditions and their unique dynamic characterization will be left for a future study. This study essentially forms a part of an effort to develop a general dynamical modeling framework that accounts for realistic manufacturing processes and their end results into the dynamic modeling of gears and other power transmission components and systems. In return, this modeling framework is expected to help reliability and thus sustainability of life critical mechanical components.

2. Dynamic model formulation

In this study, influence of indexing errors on spur gear dynamics is demonstrated via a simplified lumped parameter dynamic model as shown in Fig. 2. It is a 6 degree-of-freedom (DOF) dynamic model that assumes both gears can translate in x and y coordinates and also can rotate about their own local rotational axis represented by the z coordinates. However, the line of action (LOA) is selected such that it is coincident with the global x-axis of the inertial frame which uncouples the gear dynamics