Mechanical Systems and Signal Processing 60-61 (2015) 535–546

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

A combined method for computing frequency responses of proportionally damped systems

Baisheng Wu *, Shitong Yang, Zhengguang Li, Shaopeng Zheng

Department of Mechanics and Engineering Science, School of Mathematics, Jilin University, Changchun 130012, PR China

ARTICLE INFO

Article history:
Received 9 June 2014
Received in revised form
10 November 2014
Accepted 22 January 2015
Available online 13 February 2015

Keywords:
Frequency response
Proportional damping
Modal superposition
Model order reduction
Preconditioned conjugate gradient method

ABSTRACT

Frequency response analysis requires the evaluation of an associated function for a typically large number of frequencies. Direct method for performing these calculations is time-consuming. In this paper, a method is proposed for solving frequency responses of a mechanical system with proportional damping. The method combines modal superposition with a model order reduction. Only the modes corresponding to a frequency range which is a little bigger than that of interest are used for modal superposition. Complementary part of contribution of computed modes for frequency response is calculated by a model order reduction method. Basis vectors are obtained by applying preconditioned conjugate gradient method to a modified undamped system at the highest frequency of interest. The existing factorized stiffness matrix developed for partial eigensolutions is used as preconditioner. This computational methodology is illustrated by its applications to two frequency response problems. It is shown that the present method can remarkably reduce the CPU time required by the direct method to frequency response analysis.

1. Introduction

Frequency response analysis arises in structural dynamic problems subjected to harmonically varying loading, and plays a very important role in many applications such as model updating [1]. It requires the evaluation of a frequency response function for a wide frequency band, is thus a very challenging problem.

The direct method is the most straightforward one for solving undamped/damped frequency response problem. For such a method, a factorization of a large scale dynamic stiffness matrix is needed, forward and back substitution processes are then involved for every exciting frequency. The approach is very expensive in terms of CPU cost for large scale finite element models.

Since the frequency responses are typically desired in a wide frequency range, cost reduction methods are an advisable choice for large scale systems. Accurate and yet inexpensive evaluations of frequency response analysis is thus attractive. Many techniques have been developed in the literature for reducing computational cost of the aforementioned direct method. These approaches include the modal superposition method [2,3], modal acceleration method [4–6], hybrid expansion method [7–10], model order reduction (MOR) method [11–14], and so on. The modal superposition method and modal acceleration method are often used to extract the frequency responses. The problem is that a relatively large number

* Corresponding author.
E-mail address: wubs@jlu.edu.cn (B. Wu).

http://dx.doi.org/10.1016/j.ymssp.2015.01.018
0888-3270/

of eigenvectors and eigenvalues may be required with a relatively high precision and it is not clear how many modes should be chosen to ensure the accuracy. If these remained modes are not enough, the results obtained by both methods may be meaningless. Hybrid expansion methods [7–10] are based on modal superposition and power series expansion of the dynamic flexible matrix. However, the implementations of these methods will suffer from more computational effort, including: (a) How many modes are necessary to evaluate the frequency response accurately? (b) How many terms, levels, should be considered in the power series? (c) How do we judge that the results have the necessary accuracy? (d) How to avoid overflow or underflow which may occur in computing the corresponding coefficients of the expansion? Kim and Bennighof [15] utilized the pre-computed eigenpairs to compress the system, then used the singular value decomposition to solve the reduced system. For this method, however, computation of a large number of eigenvectors and eigenvalues is required, which is quite time-consuming. Unlike alternative cost reduction methods based on spectral or singular value decompositions, Beattie and Gugercin [16,17] illustrated that interpolatory MOR methods, such as the derivative-based Galerkin projection (DGP) method [13] and the Krylov-based Galerkin projection (KGP) method [13], are powerful tools for reducing computational cost. However, the performance of these methods depends on the location and number of the interpolation frequency points. It also depends on the number of consecutive frequency derivatives of the response function that are matched at each frequency point. Hetmaniuk et al. [13] indicated that, for the same number of basis, multi-point interpolatory MOR is more accurate than patching together one-point interpolatory reduced-order models. Meerbergen [11] and Han [12] proposed a Lanczos-based MOR method and a Krylov-based MOR method for mechanical systems with proportional damping, respectively. The limitation of these two methods is that the right-hand side vector should not have a spatial dependency on the exciting frequency. For frequency response analysis with multiple loads cases, basis vectors must be recomputed for each load case. Yet, the performance of these two methods depend on the number of basis of the projection subspace, it is not clear how many basis vectors should be chosen to ensure the accuracy. Hetmaniuk et al. [14] developed an adaptive derivative based or Krylov based interpolation MOR method, which is based on monitoring the Euclidean norm of the relative residual associated with the response to be evaluated over the frequency range of interest, but the relative residual must be evaluated repeatedly during the adaptive process.

In this paper, by combining modal superposition with a MOR, we present a new method to calculate the frequency responses over a frequency range of interest for a mechanical system with proportional damping. The method requires only the modes corresponding to a frequency range which is a little bigger than that of interest for modal superposition. In the method, the governing equation for the complementary part of contribution of computed modes for frequency response without damping are first modified into symmetric positive definite system. The factorized stiffness matrix developed for an iterative eigensolution such as Lanczos or Subspace Iteration [18] is then used as preconditioner. At the highest frequency of interest, a preconditioned conjugate gradient (PCG) method is utilized to solve the corresponding system and to obtain search directions. Numbers of basis vectors composed of these search directions for the MOR are adaptively determined. These basis vectors are then used to perform the frequency response analysis. Each evaluation requires the solution of a small system of equations to compute the vector of generalized coordinates. The most interesting use of the proposed method is to allow very fast development of approximate solutions of high quality and low computational cost. This will be demonstrated by two numerical examples of large finite element models.

2. Problem and formulas

A multi-degree-of-freedom structural system subjected to a time-harmonic external force can be written as

$$\mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{K}\mathbf{X}(t)=\mathbf{F}e^{i\omega t}, \quad \omega\in[0,\omega_R] \tag{1}$$

where $\mathbf{F}=\mathbf{F}_1+i\mathbf{F}_2$ represents the amplitude and initial phase of a n time-harmonic external input, and $\mathbf{F}_1$ and $\mathbf{F}_2$ are real vectors; ω is the excitation angular frequency; $[0,\omega_R]$ is the angular frequency band of interest; a dot denotes differentiation with respect to time t; $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are large and sparse $n\times n$ mass, damping and stiffness matrices, respectively, and $\mathbf{M}$ and $\mathbf{K}$ are symmetric positive definite. In this paper, proportional damping

$$\mathbf{C}=\alpha\mathbf{M}+\beta\mathbf{K} \tag{2}$$

is considered where α,β are two real scalars that may depend on ω.

Assume that $\mathbf{X}(t)=\mathbf{x}e^{i\omega t}$ is the solution to Eq. (1), its substitution into Eq. (1) yields

$$(\mathbf{K}+i\omega\mathbf{C}-\omega^2\mathbf{M})\mathbf{x}=\mathbf{F}, \quad \omega\in[0,\omega_R] \tag{3}$$

Every evaluation of $\mathbf{x}(\omega)$ requires solution to system of equations in Eq. (3) which is often large in order. For this reason, the direct method for solving frequency response analysis, defined here as the repeated solution of the complex algebraic system of equations implied by Eq. (3) for every frequency of interest within $[0,\omega_R]$, is usually unaffordable. Usually, the coefficient matrix $\mathbf{K}+i\omega\mathbf{C}-\omega^2\mathbf{M}$ is complex symmetric. The conjugate gradient (CG)-type method cannot be used to solve Eq. (3), since the classical CG theory guarantees convergence only for a positive-definite coefficient matrix [19–21]. For this reason, conjugate orthogonal conjugate gradient (COCG) method [22,23], conjugate orthogonal conjugate residual (COCR) method [24] or bi-conjugate gradient (Bi-CG) method [25] have to be employed for solving complex linear systems, but breakdown or stagnation may take place. Bunse-Gerstner and Stöver [26] pointed out that, CG-like methods have to be used