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Nonlocal normal modes in nanoscale dynamical systems

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ABSTRACT

This paper introduces the idea of nonlocal normal modes arising in the dynamic analysis of nanoscale structures. A nonlocal finite element approach is developed for the axial vibration of nanorods, bending vibration of nanobeams and transverse vibration of nanoplates. Explicit expressions of the element mass and stiffness matrices are derived in closed-form as functions of a length-scale parameter. In general the mass matrix can be expressed as a sum of the classical local mass matrix and a nonlocal part. The nonlocal part of the mass matrix is scale-dependent and vanishes for systems with larger lengths. Classical modal analysis and perturbation method are used to understand the dynamic behaviour of discrete nonlocal systems in the light of classical local systems. The conditions for the existence of classical normal modes for undamped and damped nonlocal systems are established. Closed-form approximate expressions of nonlocal natural frequencies, modes and frequency response functions are derived. Results derived in the paper are illustrated using examples of axial and bending vibration of nanotubes and transverse vibration of graphene sheets.

1. Introduction

Nanoscale systems, such as those fabricated from simple and complex nanorods, nanobeams [1] and nanoplates, have attracted keen interest among scientists and engineers. Examples of one-dimensional nanoscale objects include (nanorod and nanobeam) carbon nanotubes [2], zinc oxide (ZnO) nanowires and boron nitride (BN) nanotubes, while two-dimensional nanoscale objects include graphene sheets [3] and BN nanosheets [4]. These nanoscale entities or nanostructures are found to have exciting mechanical, chemical, electrical, optical and electronic properties. Nanostructures are being used in the field of nanoelectronics, nanodevices, nanosensors, nanooscillators, nanoactuators, nanobearings, and micromechanical resonators, transporter of drugs, hydrogen storage, electrical batteries, solar cells, nanocomposites and nanooptomechanical systems (NOMS). Understanding the dynamics of nanostructures is crucial for the development of future generation applications in these areas.

Experiments at the nanoscale can be difficult as many parameters need to be taken care of. On the other hand, atomistic computation methods such as molecular dynamic (MD) simulations [5] are computationally prohibitive for nanostructures with large numbers of atoms. Thus continuum mechanics is an important tool for modelling, understanding and predicting

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physical behaviour of nanostructures. Although continuum models based on classical elasticity are able to predict the general behaviour of nanostructures, they lack the accountability of effects arising from the small-scale. At small-scale the theory and laws of classical elasticity may not hold. Consequently for accurate predictions, the employability of the classical continuum models have been questioned in the analysis of nanostructures and nanoscale systems. To address this, size-dependent continuum based methods [6–9] are getting in popularity in the modelling of small sized structures as they offer much faster solutions than molecular dynamic simulations for various nanoengineering problems. Currently research efforts are undergoing to bring in the size-effects within the formulation by modifying the traditional classical mechanics. One popularly used size-dependant theory is the nonlocal elasticity theory pioneered by Eringen [10], and applied to nanotechnology by Peddieson et al. [11]. The theory of nonlocal elasticity (nonlocal continuum mechanics) is being increasingly used for efficient analysis of nanostructures viz. nanorods [12,13], nanobeams [14], nanoplates [15,16], nanorings [17], carbon nanotubes [18,19], graphenes [20,21], nanoswitches [22] and microtubules [23]. Nonlocal elasticity accounts for the small-scale effects at the atomistic level. At nanometer scales, size effects often become prominent. Both experimental and atomistic simulation results have shown a significant size-effect in the mechanical properties when the dimensions of these structures become small [24,25]. In the nonlocal elasticity theory the small-scale effects are captured by assuming that the stress at a point as a function of the strains at all points in the domain. Nonlocal theory considers long-range inter-atomic interaction and yields results dependent on the size of a body [10]. Some of the drawbacks of the classical continuum theory could be efficiently avoided and size-dependent phenomena can be explained by the nonlocal elasticity theory. A good review on nonlocal elasticity and application to nanostructures can be found in Ref [26].

Several researchers have used nonlocal theory for dynamic analysis of continuum systems such as nanorods, nanobeams and nanoplates. Nanorods have found application in energy harvesting, light emitting devices and microelectromechanical systems (MEMS). Using nonlocal elasticity, various work on mechanical behaviour of nanorods [12,13,27–29] were reported. Numerous works are seen in the literature regarding analysis (mainly structural) of nanobeams using nonlocal elasticity [26] and coupled nanobeams [14]. The work on nanobeams is related to carbon nanotubes, boron nitride nanotubes and ZnO nanowires. Nanoplate models have been used to represent two-dimensional nanostructures such as graphene sheets and BN sheets. Several works on dynamics of nanoplates using nonlocal theory are available in literature [30,31].

From the brief literature review it is clear that significant research efforts have taken place in the analysis of nanostructures modelled as a continuum. While the results have given significant insights, the analysis is normally restricted to single-structure (e.g, a beam or a plate) with simple boundary conditions and no damping. In the future complex nanoscale structures will be used for next generation nanoelectro-mechanical systems. Therefore, it is necessary to have the ability for design and analysis of damped built-up structures. The finite element approach for nanoscale structures can provide this generality. Work on nonlocal finite elements is in its infancy stage. Pisano et al. [32] reported a finite element procedure for nonlocal integral elasticity. Chang [33] studied the small scale effects on axial vibration of non-uniform and nonhomogeneous nanorods by using the theory of nonlocal elasticity and the finite element method. Narendar and Gopalakrishnan [34] used the concept of nonlocal elasticity and applied it for the development of a spectral finite element (SFE) for analysis of nanorods. Recently Adhikari et al. [35] reported the free and forced axial vibrations of damped nonlocal rods using dynamic nonlocal finite element analysis. Similar to the few works on nonlocal finite element analysis of nanorods, not many works were reported on the nonlocal finite element formulation of nanobeams (carbon nanotubes). Phadikar and Pradhan [30] have proposed basic finite element formulations for a nonlocal elastic EulerBernoulli beam using the Galerkin technique. Studies were carried out for bending, free vibration and buckling for nonlocal beam with four classical boundary conditions. Pradhan [36] updated the work of nonlocal finite element to Timoshenko beam theory and applied it to carbon nanotubes. With the finite element analysis bending, buckling and vibration for nonlocal beams with clamped–clamped, hinged–hinged, clamped–hinged and clamped-free boundary conditions were illustrated. The basic nonlocal finite elements of undamped two-dimensional nanoplates (such as graphene sheets) were reported by Phadikar and Pradhan [30]. Recently, Ansari et al. [37] developed nonlocal finite element model for vibration of embedded multi-layered graphene sheets. The proposed finite elements were based on the Mindlin-type equations of motion coupled together through the van der Waals interaction. Vibrational characteristics of multi-layered graphene sheets with different boundary conditions embedded in an elastic medium were considered.

The majority of the reported works on nonlocal finite element analysis consider free vibration studies where the effect of non-locality on the undamped eigensolutions has been studied. Damped nonlocal systems and forced vibration response analysis have received little attention. On the other hand, significant body of literature is available [38–40] on finite element analysis of local dynamical systems. It is necessary to extend the ideas of local modal analysis to nonlocal systems to gain qualitative as well as quantitative understanding. This way, the dynamic behaviour of general nonlocal discretised systems can be explained in the light of well known established theories of discrete local systems. The purpose of this paper is to make contributions in this open area.

The paper is organised as follows. In Section 2 we introduce the nonlocal finite element formulation for the axial vibration of rods, bending vibration of beams and transverse vibration of plates. Explicit expressions of element mass and stiffness matrices for the three systems are derived. Modal analysis of discrete nonlocal dynamical systems is discussed in Section 3. The conditions for the existence of classical normal modes, approximations for nonlocal frequencies and modes are proposed. In Section 4, dynamic response of damped nonlocal systems and approximation to the frequency response function are discussed. Analytical results, including the approximations of the nonlocal natural frequencies and modes, are numerically illustrated for the three systems in Section 5. In Section 6 some conclusions are drawn based on the theoretical and numerical results obtained in the paper.