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A modified complex modal testing technique for a rotating tire with a flexible ring model

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ABSTRACT

Natural frequencies, mode shapes and modal damping values are the most important parameters to describe the noise and vibration behavior of a mechanical system. For rotating machinery, however, the directivity of the propagation wave of each mode should also be taken into account. For rotating systems, this directivity can be determined by complex modal testing. In this paper, a rolling tire is represented as a flexible ring model. The limitation of application of the complex modal testing which requires two directional measurements at a certain point, which is difficult to measure in practice, has been overcome through a modified complex modal testing which requires only one directional measurements at any two points. The technique is described in detail and applied to both a numerical example and to an experimental data set of a real rotating tire.

1. Introduction

The dynamic characteristics of rolling tires have been widely studied in the literature by looking at modes and propagating waves [1–3]. Next to the natural frequencies and the damping values, the directivity of the propagation wave of each complex mode (forward and backward waves) is an important parameter for the understanding of the dynamic characteristics of rolling tires. Furthermore, an accurate estimation of the complex mode shape itself is also of great importance [4]. However, the experimental setup needed to identify the mode shapes using conventional modal testing approaches is quite complex and expensive as many points should be measured on the rotating object [1,2]. For this reason, Lee [5–7] introduced a complex modal testing (CMT) approach for rotating machinery, which makes it possible to estimate the directivity of each wave with only two responses measured at different points and two forces applied at different locations.

The equation of motion for the rotating system is composed of two directions (e.g. for the rotor/bearing system it is described in the Y and Z directions and for the flexible ring model it is described in the radial and circumferential direction). These two directional vibration measurements at a certain point are used in the conventional CMT technique. In current experimental set-up of a rotating tire, it is very difficult to measure the both directional responses (radial and circumferential directions) at a certain point. Therefore, the conventional CMT and the modified CMT that are proposed in this paper can be classified by following explanations. The modified CMT technique, which uses only one directional measurement measured at

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any two points (in this research the radial direction is used), is introduced with the flexible rotating ring model to get same results to the conventional CMT. And to introduce in a simplified manner, the conventional CMT is developed for estimating the directivity of each wave as well as the degree of anisotropy or asymmetric of rotors by using directional frequency response function (dFRF). In this technique, the dFRF is identified with two directional vibration measurements, which compose the equation of motion of a rotor/bearing system in a Cartesian coordinate, at one position. While the proposed modified complex modal testing in this study is developed based on a rotating flexible ring model. In the modified technique, the dFRFs are composed of two vibrational measurements in radial direction measured at two different positions and these are used to confirm the directivity of each wave by the difference of magnitudes between the results in the positive and negative frequency regions of the dFRFs.

In fact there are several limitations on analyzing dynamic behavior of a rotating tire with the flexible ring mode as introduced in Ref. [8]. For instance, a rolling tire is deformed by vehicle's weight during driving condition [3]. The proposed ring model, however, has always a circular configuration during rotating condition, in addition, tire includes bending mode in axial direction but ring model cannot take this into account [4]. Despite of these limitations, the reason of adopting the flexible ring model is that it is easy to confirm and to estimate the rotational effects (forward and backward waves) in the analytic FRF result. And this study does not focus on investigation of dynamic changes of a rolling tire by loading and rotating conditions but development of a method to estimate the rotational effects. In addition, it is observed that the estimated rotational effects confirmed by the proposed method are in well agreement with the confirmed results in experimental measurement within interesting frequency region. Hence, it can be concluded that the introduced flexible ring model is effective to analyze rotational effect of a tire.

In the current paper, the conventional CMT theory will be briefly introduced and two measurement conditions will also be introduced to estimate the directivity of each wave with confidence for the modified CMT technique. And the approach with these conditions will be applied on the case of a rotating tire. First the approach is applied on the numerical case of a tire ring model [9] and afterwards on measurements on a real rotating tire.

2. Complex modal testing [5]

2.1. Directional frequency response function

The equation of motion of an isotropic rotor/bearing system under complex force excitation can be written as

$$\mathbf{M_c}\ddot{\mathbf{p}}+\mathbf{C_c}\dot{\mathbf{p}}+\mathbf{K_c}\mathbf{p}=\mathbf{g} \tag{1}$$

with

$$\mathbf{M_c} = \mathbf{M}_1 - j\,\mathbf{M}_2\,,\ \mathbf{C_c} = \mathbf{C}_1 - j\,\mathbf{C}_2\,,\ \mathbf{K_c} = \mathbf{K}_1 - j\,\mathbf{K}_2$$
$$\mathbf{M}_1 = \mathbf{M}_{yy} = \mathbf{M}_{zz}\,,\ \mathbf{C}_1 = \mathbf{C}_{yy} = \mathbf{C}_{zz}\,,\ \mathbf{K}_1 = \mathbf{K}_{yy} = \mathbf{K}_{zz}$$
$$\mathbf{M}_2 = \mathbf{M}_{yz} = -\mathbf{M}_{zy}\,,\ \mathbf{C}_2 = \mathbf{C}_{yz} = -\mathbf{C}_{zy}\,,\ \mathbf{K}_2 = \mathbf{K}_{yz} = -\mathbf{K}_{zy}$$

and

$$\mathbf{p} = \mathbf{y} + j\mathbf{z}\,,\ \mathbf{g} = \mathbf{f}_y + j\mathbf{f}_z$$

In Eq. (1), vector $\mathbf{p}$ and $\mathbf{g}$ represent the complex response vector and the complex force vector which are composed of displacement vectors and external force vectors in y and z direction, with y and z describing the plane perpendicular to the axis of rotation, and the size of each vector is $N \times 1$. In addition, subscript c represents complex matrices that are composed of complex values. And the size of $\mathbf{M_c}$, $\mathbf{C_c}$ and $\mathbf{K_c}$ is $N \times N$ and the equation can be transformed into a state space form

$$\mathbf{A_c}\dot{\mathbf{w}}_\mathbf{c}+\mathbf{B_c}\mathbf{w_c}=\mathbf{F_c} \tag{2}$$

with

$$\mathbf{A_c}=\begin{bmatrix} 0 & M_c \\ M_c & C_c \end{bmatrix},\mathbf{B_c}=\begin{bmatrix} -\mathbf{M_c} & \mathbf{0} \\ \mathbf{0} & \mathbf{K_c} \end{bmatrix},\mathbf{w_c}=\begin{Bmatrix} \dot{\mathbf{p}} \\ \mathbf{p} \end{Bmatrix}\text{and } \mathbf{F_c}=\begin{Bmatrix} \mathbf{0} \\ \mathbf{g} \end{Bmatrix}$$

The size of the state matrices $\mathbf{A_c}$ and $\mathbf{B_c}$, in Eq. (2), is $2N \times 2N$ and these are composed of complex values and are non-symmetric matrices. The size of the state response vector $\mathbf{w_c}$ and the state force vector $\mathbf{F_c}$ is $2N \times 1$. If the eigenvalue problem of Eq. (2) is solved with the assumption that $\mathbf{F_c}$ is equal to $\mathbf{0}$, then the eigenvectors and adjoint eigenvectors can be obtained, and the bi-orthogonal property can be satisfied as follows

$$\begin{aligned} \mathbf{l}_{cj}^H\mathbf{A}_c\mathbf{r}_{ci} &= \delta_{ji} \\ \mathbf{l}_{cj}^H\mathbf{B}_c\mathbf{r}_{ci} &= -\lambda_i\delta_{ji} \end{aligned} \tag{3}$$

In Eq. (3), the vectors $\mathbf{r_c}$ and $\mathbf{l_c}$ represent eigenvectors (right eigenvectors) and adjoint eigenvectors (left eigenvectors), respectively, and λ represents the eigenvalues of the system, which are expressed by complex values. The real terms of λ imply energy dissipation and the imaginary terms of λ refer to the resonance frequencies of the system. Here, positive $(+)$ resonance frequencies represent propagating directivity of the wave in forward direction, while negative $(-)$ resonance