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Performance, robustness and sensitivity analysis of the nonlinear tuned vibration absorber

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ABSTRACT

The nonlinear tuned vibration absorber (NLTVA) is a recently developed nonlinear absorber which generalizes Den Hartog's equal peak method to nonlinear systems. If the purposeful introduction of nonlinearity can enhance system performance, it can also give rise to adverse dynamical phenomena, including detached resonance curves and quasiperiodic regimes of motion. Through the combination of numerical continuation of periodic solutions, bifurcation detection and tracking, and global analysis, the present study identifies boundaries in the NLTVA parameter space delimiting safe, unsafe and unacceptable operations. The sensitivity of these boundaries to uncertainty in the NLTVA parameters is also investigated.

1. Introduction

A recent trend in the technical literature is to exploit nonlinear dynamical phenomena instead of avoiding them, as is the common practice. For instance, reference [1] demonstrates a new mechanism for tunable rectification that uses bifurcations and chaos. In [2], a new strategy for engineering low-frequency noise oscillators is developed through the coupling of modes in internal resonance conditions. A cascade of parametric resonances is proposed by Strachan et al. as a basis for the development of passive frequency dividers [3].

Nonlinearity is also more and more utilized for vibration absorption [4–7] and energy harvesting [8–11]. For instance, a nonlinear energy sink (NES), i.e., an absorber with essential nonlinearity [12], can extract energy from virtually any mode of a host structure [13]. The NES can also carry out targeted energy transfer, which is an irreversible channeling of vibrational energy from the host structure to the absorber [14]. This absorber was applied for various purposes including seismic mitigation [15], aeroelastic instability suppression [16,17], acoustic mitigation [18] and chatter suppression [19]. Another recently developed absorber is the nonlinear tuned vibration absorber (NLTVA) [20]. A unique feature of this device is that it can enforce equal peaks in the frequency response of the coupled system for a large range of motion amplitudes thereby generalizing Den Hartog's equal peak method to nonlinear systems. The NLTVA is therefore particularly suitable for mitigating the vibrations of a nonlinear resonance of a mechanical system. It was also found to be effective for the suppression of limit cycle oscillations [21].

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These contributions demonstrate that the purposeful introduction of nonlinearity can enhance system performance. However, nonlinearity can also give rise to complicated dynamical phenomena, which linear systems cannot. If quasiperiodic regimes of motion can be favorable for vibration absorption with essential nonlinearity [22], they were found to be detrimental for a nonlinear absorber possessing both linear and nonlinear springs [23]. This highlights that no general conclusion can be drawn regarding the influence of quasiperiodic attractors. Detached resonance curves (DRCs), also termed isolas, are generated by the multivaluedness of nonlinear responses and may limit the practical applicability of nonlinear absorbers [24,25]. An important difficulty with DRCs is that they can easily be missed, because they are detached from the main resonance branch [5,26]. Finally, we note that DRCs were found in other applications involving nonlinearities, such as shimmying wheels [27] and structures with cyclic symmetry [28], showing the generic character of DRCs.

In view of the potentially adverse effects of the aforementioned nonlinear attractors, the main objective of the present paper is to identify boundaries in the NLTVA parameter space delimiting safe, unsafe and unacceptable operations. The sensitivity of these boundaries to uncertainty in the NLTVA parameters is also investigated. To this end, rigorous nonlinear analysis methods, i.e., numerical continuation of periodic solutions, bifurcation detection and tracking, and global analysis, are utilized. Although these methods are well-established, their combination in a single study has not often been reported in the vibration mitigation literature.

The paper is organized as follows. Section 2 briefly reviews the salient features of the NLTVA. Specifically, this section demonstrates that equal peaks in the frequency response of the coupled system can be maintained in nonlinear regimes of motion. Section 3 reveals that systems featuring a NLTVA can exhibit DRCs and quasiperiodic regimes of motion. Based on the existence and location of these attractors, regions of safe, unsafe and unacceptable NLTVA operations are defined. Section 4 studies the sensitivity of attenuation performance and of the three regions of NLTVA operation to variations of the different absorber parameters. The conclusions of the present study are summarized in Section 5.

2. Performance of the nonlinear tuned vibration absorber

The NLTVA targets the mitigation of a nonlinear resonance in an as large as possible range of forcing amplitudes. An unconventional feature of this absorber is that the mathematical form of its nonlinear restoring force is not imposed a priori, as it is the case for most existing nonlinear absorbers. Instead, we fully exploit the additional design parameter offered by nonlinear devices, and, hence, we synthesize the absorber's load–deflection curve according to the nonlinear restoring force of the primary structure.

The dynamics of a Duffing oscillator with an attached NLTVA, as depicted in Fig. 1, is considered throughout this study:

$$\begin{aligned} &m_1\ddot{x}_1+c_1\dot{x}_1+k_1x_1+k_{nl1}x_1^3+c_2(\dot{x}_1-\dot{x}_2)+g(x_1-x_2)=F\cos\omega t \\ &m_2\ddot{x}_2+c_2(\dot{x}_2-\dot{x}_1)-g(x_1-x_2)=0 \end{aligned} \tag{1}$$

where $x_1(t)$ and $x_2(t)$ are the displacements of the harmonically forced primary system and of the NLTVA, respectively. The NLTVA is assumed to have a generic smooth restoring force $g(x_1-x_2)$ with $g(0)=0$. In order to avoid important sensitivity of absorber performance to forcing amplitude, it was shown in Ref. [20] that the function $g(x_1-x_2)$ should be chosen such that the NLTVA is a 'mirror' of the primary system. More precisely, besides a linear spring, the NLTVA should possess a nonlinear spring of the same mathematical form as that of the nonlinear spring of the primary system. For instance, if the nonlinearity in the primary system is quadratic or cubic, the NLTVA should possess a quadratic or a cubic spring, respectively. To mitigate the vibrations of the Duffing oscillator, a NLTVA with linear and cubic stiffnesses is therefore considered, i.e., $g(x_1-x_2)=k_2(x_1-x_2)+k_{nl2}(x_1-x_2)^3$, and the governing equations of motion become

$$\begin{aligned} &m_1\ddot{x}_1+c_1\dot{x}_1+k_1x_1+k_{nl1}x_1^3+c_2(\dot{x}_1-\dot{x}_2)+k_2(x_1-x_2)+k_{nl2}(x_1-x_2)^3=F\cos\omega t \\ &m_2\ddot{x}_2+c_2(\dot{x}_2-\dot{x}_1)+k_2(x_2-x_1)+k_{nl2}(x_2-x_1)^3=0 \end{aligned} \tag{2}$$

In view of the effectiveness of the equal-peak method [30,31] for the design of linear tuned vibration absorbers (LTVA) attached to linear host structures, an attempt to generalize this tuning rule to nonlinear absorbers attached to nonlinear host structures was made in reference [20]. The first step was to impose equal peaks in the receptance function of the underlying linear system using the formulas proposed by Asami et al. [29]

$$k_2^{opt}=\frac{8\epsilon k_1\left[16+23\epsilon+9\epsilon^2+2(2+\epsilon)\sqrt{4+3\epsilon}\right]}{3(1+\epsilon)^2(64+80\epsilon+27\epsilon^2)},\quad c_2^{opt}=\sqrt{\frac{k_2m_2(8+9\epsilon-4\sqrt{4+3\epsilon})}{4(1+\epsilon)}} \tag{3}$$

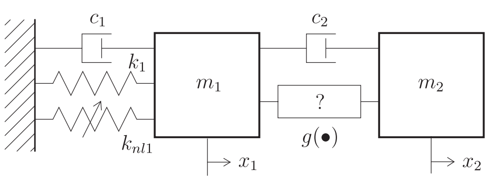

Fig. 1. Schematic representation of an NLTVA attached to a Duffing oscillator.