Mechanical Systems and Signal Processing 60-61 (2015) 853–865

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

A hybrid parameter identification method based on Bayesian approach and interval analysis for uncertain structures

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ARTICLE INFO

Article history:
Received 22 May 2014
Received in revised form
26 November 2014
Accepted 12 February 2015
Available online 4 March 2015

Keywords:
Inverse problem
Parameter identification
Bayesian approach
Interval analysis
Measurement noise
Model uncertainty

ABSTRACT

The hybrid inverse method based on Bayesian approach and interval analysis is presented for parameter identifications under uncertainty, which can deal with both measurement noise and model uncertainty. The measurement noise from an experiment may be described by a set of random variables, obeying a certain probability distribution. The each uncertain parameter of a structure model may be treated as an interval, and only the bounds of the uncertainty are needed. Because of the existence of the interval parameters, a posterior probability density distribution strip enclosed by two bounding distributions is then formed, rather than a single distribution that we usually obtain through the Bayesian identification for a deterministic structure. Using an interval analysis method, a structure response with small uncertainty levels can be approximated as a linear function of the interval parameters. A monotonicity analysis is adopted for marginal posterior distribution transformation, through which effects of the interval parameters on the posterior distribution strip can be well revealed. Based on the monotonicity analysis, finally, the mean estimates and confidence intervals of the unknown parameters are identified from the posterior distribution strip. Three numerical examples are investigated, and fine numerical results are obtained.

1. Introduction

In general, inverse problems are defined as problems to determine the input through the given output, in contrast with forward problems which are concern with the output response from the known input. Parameter identifications, which focus on determination of the unknown parameters in a specific model from a series of indirect measurement data, are considered as one of typical inverse problems. To successfully solve this type of inverse problem, its ill-posed nature due to universal existence of uncertainty in practical applications should be treated. Some strategies, such as probability [1–4], interval [5] and combined method [6] have been proposed to deal with this issue; however, this problem remains still a challenging one. Parameter identifications under uncertainty are widely open to develop effective inverse methods.

According to their location in the process of identifying unknown parameters for a practical application, the existing uncertainties can be roughly divided into two different types in this paper. The one is that the observed data are mostly corrupted by the measurement errors, namely measurement noise; the other is that a structure model is often imprecise

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http://dx.doi.org/10.1016/j.ymssp.2015.02.009
0888-3270/

due to simplification by hypotheses, namely model uncertainty. To obtain reasonable identified results, therefore, these two types of uncertainty should be examined.

Recently, much attention is paid on how to treat measurement noise. This is because the response of a structure would never be perfectly measured corresponding to the actual response due to limitation of the instrument precision and/or complexity of the working conditions. Among the available strategies, Bayesian approach [7] is considered more promising one to deal with measurement noise. Its main advantage comes from combining a likelihood function with prior parameter knowledge to produce corresponding posterior probability density function (PPDF). The PPDF encapsulates all available information about the identified parameters. Numerous statistical variables, such as marginal distributions, means, and confidence intervals for each unknown parameter, can be calculated. Hence, the uncertainty that propagate from the measurement noise to the resulting parameters can be easily quantified. To this end Bayesian approach as well as its modified forms has received considerable attention in various inverse problems in engineering, such as material characterization [8,9], inverse design [10–12], uncertainty propagation [13], and damage detection [14–16]. While this class of method met with some success, the influence of the uncertain model on the identified results should be deeply studied.

Bayesian approach is a typical model-based method and thus the final identified results heavily depend on the predicted quality of the forward model. By virtue of their effective predictive ability, the finite element method (FEM), finite difference method (FDM), and mesh free method (MFM), etc. are often used to construct a forward model for the inverse analysis. However, due to limitation of the model itself, the forward model hardly provides a reliable response especially for complex structures [17]. The main limitation is model uncertainty that should include geometry, material properties, boundary, and initial conditions, etc. To date, the influence of model uncertainty on the identified results has not been reported as a main issue in parameter identification problems. Liu et al. [18] presented a combined method based on interval analysis and regularization method to identify the dynamic load for uncertain structures. Zhang et al. [19] treated both the measurement noise and model uncertainty as random variables in probability framework with Bayesian perspective for a force reconstruction problem.

In the present work, on account of the both presence of measurement noise and model uncertainty, we investigate parameter identifications from a Bayesian framework. Same as Ref. [19], our study views the measurement noise as a random variable, since it is inherently aleatory. However, the model uncertainty is considered as an interval [20] by epistemic uncertainty due to lack of the knowledge. This would have two main advantages. First, interval analysis method [21] can make uncertainty structure analysis more convenient, as it needs only the bounds of the uncertain parameters. Second, significant computational gains can be realized, since it avoids the time-consuming calculations by using the cheap interval analysis. Thus, based on Bayesian approach and interval analysis, a hybrid inverse framework can be established for identifying the unknown parameters and jointly quantifying measurement noise and model uncertainty.

2. Statement of the problem

For a general parameter identification problem with measurement noise and model uncertainty, the forward model can be expressed as:

$$\mathbf{Y}=\mathbf{F}(\mathbf{Q},\mathbf{m})+\mathbf{e} \tag{1}$$

where $\mathbf{Y}$ is a vector of output that represents the measured response quantities (e.g. displacements, stresses, and natural properties). $\mathbf{m}=[m_1,m_2,\ldots,m_l]^T$ denotes a vector that collects all uncertain parameters, such as the material properties, loads, and boundary conditions. And $\mathbf{Q}=[Q_1,Q_2,\ldots,Q_M]^T$ is an unknown parameter vector, which would be determined. l and M denote the number of the uncertain and unknown parameter, respectively. $\mathbf{F}$ is defined as the system matrix of functions representing the translation process from input to output. And $\mathbf{e}$ is a noise vector.

From measurement we have structural response $\mathbf{Y}$ at specific location giving the K-point time-series with noise:

$$Y_{nk}=f_{nk}+e_{nk},\quad k=1,2,\ldots,K,\quad n=1,2,\ldots,N \tag{2}$$

where $y_{nk}\in\mathbf{Y}$ denotes the response of the some variable "n" on a structure system at discrete time "k". f_{nk} is a noise-free response and e_{nk} is a sequence of independent samples drawn from a zero-mean Gaussian distribution with a variance σ^2. K and N denote the number of the time point and the measured response quantity, respectively. Note that only Gaussian distribution is used to describe $\mathbf{Y}$ without considering non-Gaussian distribution (Weibull, Lognormal and Exponential etc.) and imprecise distribution (due to lack of samples) for convenience of analysis in this work.

Fig. 1 shows illustration of parameter identification for uncertain structures with measurement noise. Random variables are used to model the noisy measurement data $\mathbf{Y}$ in experiment, which obeys a probability distribution with a certain mean and a variance. Because of uncertainty propagation, it is clear that the unknown parameters $\mathbf{Q}$ would also follow a certain probability distribution. Intervals are used to describe the uncertain parameters $\mathbf{m}$ in a forward model. Thus, the response at one receiving point on the structure is also an interval at each time point, and on the whole time history there exist two response bounds. All of the possible responses caused by the model uncertainty will be within these two bounds. In Fig. 1, the part enclosed by the rectangular framework can be regarded as an interval forward model that outputs an interval response vector $[\mathbf{Y}^L,\mathbf{Y}^R]$, for an input unknown parameter vector $\mathbf{Q}$ with the uncertain parameter vector $\mathbf{m}$. In this situation, in order to identify the mean estimates and confidence intervals of the unknown parameters $\mathbf{Q}$, an appropriate inverse method should be developed.