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# Error bounds for joint detection and estimation of multiple unresolved target-groups

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According to random finite set (RFS) and information inequality, this paper derives an error bound for joint detection and estimation (JDE) of multiple unresolved target-groups in the presence of clutters and missed detections. The JDE here refers to determining the number of unresolved target-groups and estimating their states. In order to obtain the results of this paper, the states of the unresolved target-groups are modeled as a multi-Bernoulli RFS first. The point cluster model proposed by Mahler is used to describe the observation likelihood of each group. Then, the error metric between the true and estimated state sets of the groups is defined by the optimal sub-pattern assignment distance rather than the usual Euclidean distance. The maximum a posteriori detection and unbiased estimation criteria are used in deriving the bound. Finally, we discuss some special cases of the bound when the number of unresolved target-groups is known a priori or is at most one. Example 1 shows the variation of the bound with respect to the probability of detection and clutter density. Example 2 verifies the effectiveness of the bound by indicating the performance limitations of the cardinalized probability hypothesis density and cardinality balanced multi-target multi-Bernoulli filters for unresolved target-groups. Example 3 compares the bound of this paper with the (single-sensor) bound of [4] for the case of JDE of a single unresolved target-group. At present, this paper only addresses the static JDE problem of multiple unresolved target-groups. Our future work will study the recursive extension of the bound proposed in this paper to the filtering problems by considering the group state evolutions.



## 1. Introduction

An unresolved target-group [1,2] (also called a 'point target cluster' in [3], pp. 432–433) is a cluster of two or more individual targets, which cannot be completely resolved due to finite sensor resolution, type of sensor and target-to-sensor geometry. In other words, by treating a cluster of indistinguishable point targets as an entire object (especially when the individual targets have a coordinated behavior according to high interdependency, it is more reasonable to treat them as an entire object), they are idealized as an unresolved target-group. In general, an unresolved target-group may generate multiple measurements at each scan. As shown in Fig. 1, although the number of detections generated by a group depends on the number of individual targets in the group, they are usually different due to the effect of sensor resolution, missed detection and clutter [4].

It should be noted that the unresolved target-group is significantly different from the extended target ([3], pp. 427–430), although both of them may generate multiple measurements at a scan. The extended target arises because the resolution of a sensor is higher than the spatial extent of a target. In such a case, the sensor is able to receive more than one measurement from different corner reflectors (or measurement sources) of the target. The locations of these measurement sources depend on the shape of the target, target-dependent properties (i.e., the nature of the surface) and the target-to-sensor geometry [5]. A comparison between extended target and unresolved target-group is shown in Fig. 2 [5].

The problem of joint detection and estimation (JDE) arises from a host of applications in defense and surveillance [6], where the number of targets is unknown and the sensor may receive the measurements randomly generated from targets or clutters. There is no information about which are the measurements of interest or which are the clutters. The aim of JDE is to determine the number of the targets and estimate their states using prior information about the targets as well as a sequence of measurements in the presence of clutters, missed detections and association uncertainties. Before about 1997, the study of JDE problem is only restricted

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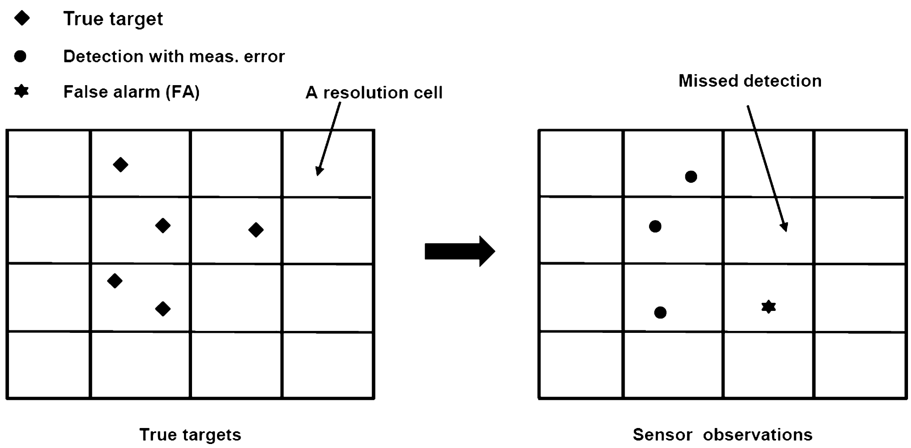


**Fig. 1.** A sensor observes five point targets. Two targets lie in the same resolution cell and are not resolved. The sensor produces detections with measurement errors. One target is not detected due to missed detection and a false alarm appears due to clutter. We can say that the five point targets form an unresolved target-group.

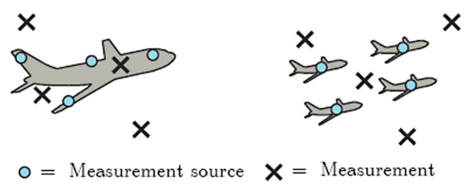


**Fig. 2.** Extended target (left) and unresolved target-group (right).

to point targets [7–9]. But in recent years, the JDE of multiple unresolved target-groups has attracted extensive attention and many approaches have been proposed [10–23] for it.

Obviously, it is very necessary to find an error (lower) bound to assess the achievable performance of the JDE algorithms for unresolved target-groups. The existing performance evaluation methods based on the Cramér–Rao lower bound (CRLB) [24–26] are not applicable for such a JDE algorithm since CRLB only considers the estimation error but not the detection error. Within the random finite set (RFS) framework, Rezaeian and Vo [27] derived the JDE error bound for a point target in the presence of clutters and missed detections. We extended the bound to the case of an unresolved target-group in [4]. However, compared with the case of a single object, it is significantly different and much more difficult in determining the object number and defining the error metric for the case of multiple objects. As a result, there have been no conclusions on JDE error bound for multiple unresolved target-groups till now.

By the use of the RFS and information inequality ([28], pp. 169–171, 186), this paper proposes an error bound for JDE of multiple unresolved target-groups when clutters and missed detections simultaneously exist. In order to obtain the results of this paper, we first model the states of multiple unresolved target-groups as a multi-Bernoulli RFS [29]. The point cluster model based on the concept of continuous individual target number is used to describe the observation likelihood function of the unresolved target-group as in ([3], pp. 437–440). Since the JDE error is the average distance between the true and estimated sets of group states, the usual Euclidean distance for random vectors cannot be applied here. To overcome this, the optimal sub-pattern assignment (OSPA) distance proposed in [30] for random sets is used to measure the JDE error. Maximum a posteriori (MAP) detection and unbiased estimation criteria are used in deriving the bound. Then the proposed bound is discussed in two special cases where the number of the unresolved target-groups is known a priori or is at most one. Especially in the second case, we show that under certain conditions, the bound of this paper coincides with the (single-sensor) bound in [4], which is proposed for JDE of a single unresolved target-group. Finally, three numerical examples are presented using simulated data. Example 1 shows the variation of the bound with respect to probability of detection and clutter density. Example 2 verifies the effectiveness of the bound by indicating the performance limitations of the unresolved target-group cardinalized probability hypothesis density (U-CPHD) [20] and unresolved target-group cardinality balanced multi-target multi-Bernoulli (U-CBMeMBer) [21] filters. Example 3 compares the bound of this paper with the (single-sensor) bound of [4] for the case of JDE of a single unresolved target-group.

In the current set up of this paper, our attention is restricted to the static JDE problem of multiple unresolved target-groups. Our future work will study the recursive extension of the bound proposed in this paper to the filtering problems by considering the group state evolutions.

The rest of the paper is organized as follows. Section 2 presents the background and related work for deriving our results. Section 3 describes the JDE problem of multiple unresolved target-groups based on RFS. In Section 4, we present a JDE error bound for unresolved target-groups and discuss two special cases of the bound when the number of the groups is known or is at most one. We present three numerical examples in Section 5. Conclusions and future work are given in Section 6. Relevant mathematical proofs of these conclusions are attached in Appendices A and B.

## 2. Background and related work

In order to derive the results of this paper, some necessary mathematical foundations are first presented in this section.

### 2.1. Set integral

For any real-valued function $\eta(Y)$ of a finite-set variable $Y$, its set integral is ([31], pp. 141–144)

$$\int \eta(Y)\delta Y = \sum_{n=0}^{\infty}\frac{1}{n!}\int_{\mathcal{Y}_n}\eta(Y_n)d\boldsymbol{y}_1\cdots d\boldsymbol{y}_n$$
$$= \eta(\emptyset) + \sum_{n=1}^{\infty}\frac{1}{n!}\int_{\mathcal{Y}_n}\eta(Y_n)d\boldsymbol{y}_1\cdots d\boldsymbol{y}_n, \tag{1}$$

where $\int\cdot\delta Y$ denotes the operator of a set integral, $Y_n = \{\boldsymbol{y}_i\}_{i=1}^{n} \subseteq \mathcal{Y}_n$ denotes the $n$-points set (that is, the cardinality of the set $Y_n$