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Unified low-complexity decision feedback equalizer with adjustable double radius constraint [☆]

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ABSTRACT

The maximum likelihood sequence estimation (MLSE) is an optimal equalization method to suppress Inter-Symbol Interference (ISI) in communication and storage systems. The Viterbi Algorithm (VA) provides an exact solution of MLSE. To reduce the complexity of VA, MLSE-DFE, which combines the VA within a decision feedback equalizer (DFE), is widely used in practical designs; however, the computing complexity is still too high. In this paper, we propose the SDVA-DFE, a unified DFE combining the concept of sphere detector (SD) and VA. The computing complexity of the SDVA-DFE can be reduced by proposed double radius constraints, upper radius (UR) and lower radius (LR). By adjusting the values of the two radiuses, the SDVA-DFE also provides a trade-off between performance and complexity. Simulation shows that this method is suitable for high-order modulation and long-length channel impulse response. When applied to a Lorentzian channel and channels of different eigenvalue spread, the SDVA-algorithm can reduce the complexity by over 90% at high SNR compared with MLSE-DFE.

1. Introduction

The maximum likelihood sequence estimation (MLSE), implemented via *Viterbi Algorithm* (VA) [1–3], is an optimal equalization method to solve the ISI problem [4], as shown in Fig. 1(a). However, the computing complexity of VA is proportional to the size of the signal set and grows exponentially with the length of channel memories. Many researchers [5–8] have aimed at low complexity and high performance design compared with MLSE. Recently, Jie Luo [9] mixed the sphere-constrained to Viterbi algorithm for convolutional decoder. Vikalo, *et al.* [10] proposed a similar idea for ISI channel, which reduces the computing complexity via *sphere decoding* (SD) algorithm [11,12], as shown in Fig. 1(b). This method achieves optimal performance with short-length inputs, but fails in the case of continuous inputs. In other words, this equalizer is not a practical design.

In contrast, the *decision feedback equalizer* (DFE) provides low-complexity approaches [13–15]. Hence, to further reduce the computing complexity of VA, MLSE-DFE [16,17] combined *decision-feedback* (DF) methods with Viterbi algorithm. This technique is also known as the *decision-feedback sequence estimator* (DFSE) [6]. Fig. 1(c) reveals that the MLSE-DFE uses the MLSE detector instead of a slicer in a normal DFE. In addition, the feedforward filter (FFF) is used as a *whitening matched filter* (WMF) [18] and the feedback filter (FBF) words as the *decision-feedback* to merge the data paths from VA earlier. The performance is much better than normal DFE, and the computing complexity of VA is significantly reduced. Hence, this structure is widely used in practical designs. However, the main drawback is that the complexity grows exponentially with the lengths of channel memories.

In addition, many researches [19–25] have demonstrated many complexity reduction approaches. Soft-Threshold-Based Multilayer Decision Feedback Equalizer (STM-DFE) [26] defines a reliable region (RR) to DFE. As shown in Fig. 1(d), when the DFE output is not in the reliable region, the decision is not made instantly. Instead, the log-likelihood ratio (LLR) is fed to FBF, and the process is repeated continually until the DFE output becomes reliable. As a result, the computing complexity is reduced, but the performance is sacrificed. More importantly, the reliable region is difficult to be designed at high-stage STM-DFE.

As discussed before, three approaches can be used to achieve low computing complexity: (1) removing impossible data paths (SD algorithm) (2) merging residual data paths (DF, DFSE or MLSE-DFE), and (3) selecting the best reliable data paths (RR, STM-DFE). Table 1 summarizes the pros and cons of previous works. Also, the architectures of these works are different. In this paper, we propose *the sphere detection-Viterbi algorithm in DFE* (SDVA-DFE) that

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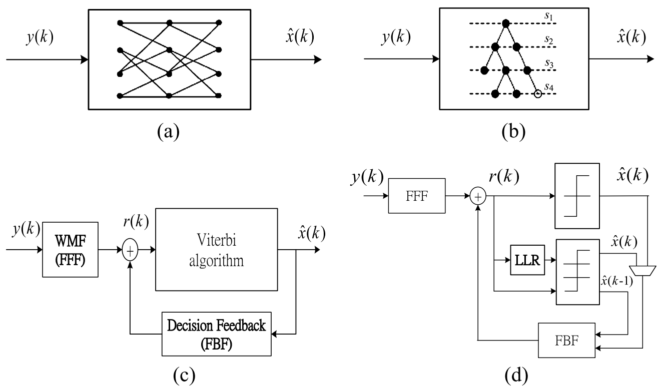

Fig. 1. The methods to reduce ISI channels in communication. (a) The Viterbi algorithm [1]. (b) Combination of Viterbi algorithm and sphere decoding [10]. (c) MLSE-DFE [16,17]. (d) Reliable region technique (STM-DFE) [26].

Table 1
Comparisons of existing works.

	SD	DF	RR	Pros and Cons
Vikalo, *et al.* [10]	◎			Highest performance, fail with continuous inputs
MLSE-DFE [16,17]		◎		Better performance, higher complexity
STM-DFE [26]		◎	◎	Not good performance, difficult high-stage extension
Proposed SDVA-DFE	◎	◎	◎	Adjustable performance, lower complexity, never fail, and easy high-stage extension

can unify the three approaches. Our proposed system, shown in Fig. 2(a), is comprised of the three techniques as follows:

1) *Double radius algorithm/constraints:* In Fig. 2(a), the radius controller provides double radius constraints to the detector. These radius constraints are the upper and lower radiuses, denoted by d_U and d_L respectively. As shown in Fig. 2(b), the upper radius, with a function similar to the SD algorithm, restricts the searching algorithm to only search the lattice points lying within it. On the other hand, the lower radius, similar to the RR technique, early terminates the searching algorithm when any lattice point lies within it. Hence, the proposed radius constraints aim to reduce the computing complexity by restricting the search region.
2) *Combining double sphere constraints with Viterbi algorithm (SDVA) in DFE:* We develop the SDVA-algorithm to achieve the maximum likelihood solution. Fig. 2(c) presents the implementation of SDVA detector based on the Viterbi trellis structure with decision-feedback, which merges the residual data paths earlier. As we control the new trellis implementation via double sphere constraints, the proposed algorithm unifies the three approaches in a single structure and considerably reduces the searching complexity.
3) *A simple operating condition of SDVA detector:* We present a simple operating condition for the SDVA detector. Under this condition, the SDVA-algorithm only searches the lattice points between the upper radius and the lower radius instead of the whole lattices points. Therefore, the ring region between the upper radius and the lower radius is defined as "radius margin".

Our SDVA-DFE can unify the three features (SD, DF and RR). The computing complexity can be reduced by the proposed double radius constraints, upper radius (UR) and lower radius (LR). Moreover, by adjusting the values of two radiuses, the SDVA-DEF provides a trade-off between performance and complexity. In brief, the characteristics of the SDVA-DFE are adjustable performance, low computing complexity, and easy high-stage extension.

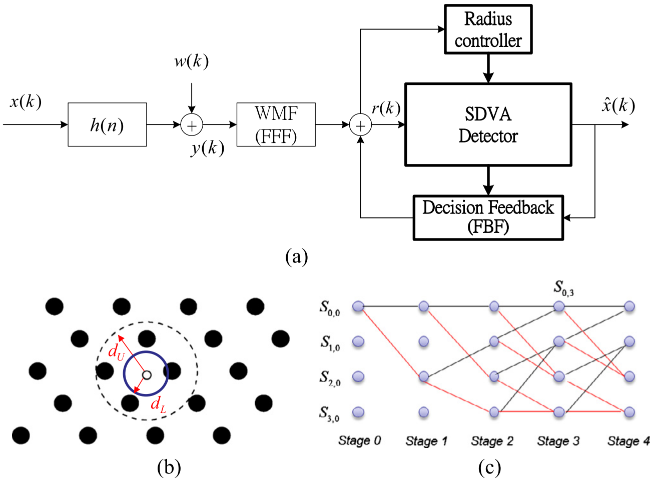

Fig. 2. The communication system and functional block of SDVA-DFE. (a) Discrete-time channel model and the block diagram of the SDVA-DFE. (b) Proposed adjustable double radiuses in radius controller. (c) Proposed structure of the SDVA detector.

The remainder of the paper is organized as follows. In Section 2, we give some background knowledge about Sphere Decoding, MLSE-DFE and the threshold value derived in [26]. Section 3 presents the proposed SDVA-DFE algorithm. The architecture of SDVA-DFE is shown in Section 4. Section 5 shows the simulation results and Section 6 concludes this paper.

2. Review of the background

The existing works are reviewed in this section. First, we give some notations and the signal model of the normal DFE, which are defined as follows. The communication system adopted in this paper is similar to that shown in Fig. 2(a).

- $x(k)$ is the transmitted signal.
- $h(n)$ is the equivalent discrete time channel impulse response, which is linear and band-limited.
- $w(k)$ is *Additive White Gaussian Noise* (AWGN).
- $y(n)$ is the channel output.
- $r(n)$ is the output of the DFE and is expressed as:

$$r(n) = \sum_{m=0}^{N_b-1} b_m y(n-m) - \sum_{m=1}^{N_a} a_m \hat{x}(n-m), \tag{1}$$

where

- $\hat{x}(n)$ is the detected signal of x_n.
- a_k is the k-th tap weight of the FBF.
- b_k is the $(k+1)$-th tap weight of the FFF (WMF).
- N_a is the tap length of the FBF.
- N_b is the tap length of the FFF (WMF).

2.1. Sphere detector [11,12]

For a MIMO system with N_T transmitting antennas and N_R receiving antennas, the received signal can be expressed as:

$$\mathbf{y} = \mathbf{Hx} + \mathbf{w}, \tag{2}$$

where $\mathbf{y} \in C^{N_R \times 1}$ is the received signal vector, $\mathbf{x} \in \psi^{N_T} \subset C^{N_T \times 1}$ is the transmitting symbol vector, and $\mathbf{w} \in C^{N_R \times 1}$ is the AWGN vector. The channel matrix $\mathbf{H} \in C^{N_R \times N_T}$ contains complex channel fading coefficients and is known before signal detection. Sphere detector (SD) can find the ML detection of $\mathbf{x}$ in the constellation