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Robust Bounding Ellipsoidal Adaptive Constrained least-squares algorithm and its performance analysis ☆

Sheng Zhang, Jiashu Zhang

Sichuan Province Key Lab of Signal and Information Processing, Southwest Jiaotong University, Chengdu, 610031, China

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ABSTRACT

In this paper, the robust Bounding Ellipsoidal Adaptive Constrained least-squares (BEACON) algorithm based on new set-membership error bounds is proposed to improve robustness and steady-state misalignment in impulsive noise environments. The expressions for the steady-state excess mean square error (EMSE) of the proposed robust BEACON are obtained. Simulations on system identification and double-talk scenarios show that the robust BEACON algorithm considerably outperforms the traditional BEACON algorithm in the presence of impulsive noise and the theoretical predictions are in good agreement with simulation results.

1. Introduction

Adaptive filters are frequently adopted in many applications such as system identification, adaptive control and adaptive echo cancellation (AEC), interference suppression in industrial and biomedical engineering [1,2]. Whenever a large number of coefficients are required, especially for echo cancellation, it diminishes the usefulness of the adaptive algorithm owing to increased complexity. Several schemes for reducing the computational complexity of adaptive filters have emerged. Partial-coefficient-update techniques update a subset of the adaptive filter coefficients [3]. "A potential drawback of partial updates is the reduction in convergence speed" [4]. By maintaining a bound on the output error magnitude of an adaptive filter, set-membership (SM) adaptive filter coefficients are only updated when the a priori error magnitude exceeds a predetermined threshold. It reduces the power consumption and computational complexity. The objective of a SM filter is to estimate set membership function, which defines the SM filter's performance specification [5–7].

The optimal bounding ellipsoids (OBE) algorithms approximate the membership function by tightly outer-bounding it with ellipsoids in the associated parameter space [7–9] that are well-established SM adaptive filter algorithms. A thorough review of the OBE family can be found in [10]. Among all the OBE algorithms, the BEACON algorithm [7,11,12] is particularly interesting since it is computationally more efficient than other OBE algorithms and because of its simple but efficient innovation check and the optimal weight calculation processes. However, as for the recursive least-squares (RLS) algorithm, the performance of the traditional BEACON algorithm is affected by outliers in the error signal samples that can be brought about by impulsive-noise interference [13]. Recently, several algorithms of the normalized least mean squares (NLMS), affine projection algorithm (APA) and Quasi-Newton (QN) in the SM filter framework that are robust with respect to outliers can be found in [14–17]. Some recursive least M-estimate algorithms that are robust with respect to outliers can be found in [18].

In this paper, we propose the robust BEACON algorithm in impulsive noise environments. Suggested by the basic idea of [16, 17], the robust BEACON algorithm is presented to enhance the robustness against impulsive noise, which contains two bounds. One bound is active during the transient state, and the other is used in steady-state and in the presence of impulsive noise. Thus, it is robust with respect to outliers brought about by impulsive noise and it also fast tracks sudden system disturbances. Compared to the BEACON, a reduced steady-state misalignment in the robust BEACON is achieved. The second contribution of this work is the analysis of steady-state performance of the proposed robust BEACON with fixed set-membership low bound. Using the energy conservation approach [19–22], the resulting model accurately predicts the proposed algorithm steady-state behavior and reveals the effects of the outlier detection threshold (ODT) parameter. In addition, the probability of the update (PU) is considered, which suggests that the proposed algorithm possesses higher PU than the BEACON. Finally, experiments in system identification and double-talk

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E-mail addresses: zhangsheng@my.swjtu.edu.cn (S. Zhang), jszhang@home.swjtu.edu.cn (J. Zhang).

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scenarios are conducted to evaluate the performance of proposed algorithm.

The rest of the paper is organized as follows. In section 2, a brief review of the SM adaptive filtering and BEACON is presented. Section 3 describes the proposed robust BEACON algorithm. Performance analysis of the proposed algorithm is carried out in Section 4. Section 5 contains simulation results and conclusions are drawn in Section 6.

2. Set-membership filter

Consider the case of system identification problem. The unknown coefficients and input signal at time instant n are denoted by $\mathbf{w}_{opt} = [w_0, w_1, \ldots, w_{L-1}]^T$ and $\mathbf{x}_n = [x_n, x_{n-1}, \ldots, x_{n-L+1}]^T$, respectively, where L is the filter length. The observed desired signal is assumed to be corrupted by an additive noise:

$$y_n = \mathbf{w}_{opt}^T \mathbf{x}_n + \upsilon_n,$$

where υ_n is the additive background noise. The estimated error between the output of unknown system and of the adaptive filter is

$$e_n = y_n - \mathbf{x}_n^T \hat{\mathbf{w}}_{n-1},$$

where $\hat{\mathbf{w}}_n = [w_{n,0}, w_{n,1}, \ldots, w_{n,L-1}]^T$ denotes the adaptive filter tap-weights at time instant n. The SM criterion corresponds to finding $\hat{\mathbf{w}}$ that satisfies the *feasibility set*

$$\Theta = \bigcap_{any\ n} \left\{ \hat{\mathbf{w}} \in R^L : \left| y_n - \mathbf{x}_n^T \hat{\mathbf{w}} \right| \le \gamma \right\}$$

where γ is a pre-specified bound. The *constraint set* that contains all vectors $\hat{\mathbf{w}}$ satisfying the error bound at time instant n, is defined as

$$H_n = \left\{ \hat{\mathbf{w}} \in R^L : \left| y_n - \mathbf{x}_n^T \hat{\mathbf{w}} \right| \le \gamma \right\}$$

Hence, the SM adaptive filtering algorithm corresponds to finding a *membership set* ψ_n at time instant n that satisfies

$$\psi_n = \bigcap_{i=1}^{n} H_i$$

where the feasibility set Θ is a subset of ψ_n.

The weights of the BEACON algorithm [7,11] are updated as follows

$$V_n = V_{n-1} - \frac{\lambda V_{n-1} \mathbf{x}_n \mathbf{x}_n^T V_{n-1}}{1 + \lambda \mathbf{x}_n^T V_{n-1} \mathbf{x}_n} \tag{1}$$

$$\hat{\mathbf{w}}_n = \hat{\mathbf{w}}_{n-1} + \lambda V_n e_n \mathbf{x}_n \tag{2}$$

where V_n is the inverse input autocorrelation matrix, the forgetting factor is

$$\lambda = \begin{cases} \frac{1}{G_n}\left(\frac{|e_n|}{\gamma} - 1\right) & \text{if } |e_n| > \gamma \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

and

$$G_n = \mathbf{x}_n^T V_{n-1} \mathbf{x}_n.$$

However, the performance of the BEACON algorithm deteriorates in the presence of outliers in the error signal samples that can be brought about by impulsive noise interference.

3. Proposed robust BEACON algorithm

In order to mitigate the effect of impulsive noise, two versions of robust BEACON algorithm are presented in this section, one with fixed set-membership low bound and the other with time-varying set-membership low bound.

3.1. Proposed robust BEACON with fixed set-membership low bound (robust BEACON-FB)

In order to achieve the robust performance against impulsive noise, the set-membership error bound (SMEB) γ in the BEACON is chosen as

$$\gamma_n = \begin{cases} \frac{e_n^2}{\nu\theta_n + |e_n|} & \text{if } |e_n| > \xi\theta_n \\ \gamma_c & \text{otherwise} \end{cases} \tag{4}$$

where we usually choose $0 < \nu < 1$ to obtain low steady-state misalignments, $\gamma_c = \sqrt{5\sigma_\upsilon^2}$ (named set-membership low bound) [5], ξ (named outlier detection threshold, ODT) is chosen to be less than $\sqrt{5}$ in order to ensure that $\xi\theta_n \le \gamma_c$ at steady-state (a similar condition is used in [16]), θ_n is chosen to be $\theta_n = \hat{\sigma}_n$, which is the impulsive-free estimation of $E[|e_n|]$ and is estimated as

$$\hat{\sigma}_n = \beta_1 \hat{\sigma}_{n-1} + (1 - \beta_1) median(\psi_n) \tag{5}$$

where β_1 is a forgetting factor, $median(\cdot)$ is the median operator and

$$\psi_n = \left[|e_n|, |e_{n-1}|, \ldots, |e_{n-N+1}| \right]^T$$

where N is the length of the estimation window.

Remark 1. Note that the γ_n in (4) is different from that used in the robust QN and SMAPA in [16,17]. The SMEB in [17] was chosen as

$$\gamma_n = \begin{cases} |e_n| - \nu\theta_n & \text{if } |e_n| > \theta_n \\ \gamma_c & \text{otherwise} \end{cases} \tag{6}$$

where $\theta_n = Q\sigma_{1,k}$ with $1.86 < Q < 1.98$, $\sigma_{1,k}^2$ is the impulsive-free estimation of $E[e_n^2]$. When the impulsive noise occurs at time instant n, using (4) and (6) in (3) yields $\lambda G_n = \nu\theta_n/|e_n|$, $\nu\theta_n/(|e_n| - \nu\theta_n)$, respectively. For $\nu\theta_n \ll |e_n|$, $\nu\theta_n/|e_n| \approx \nu\theta_n/(|e_n| - \nu\theta_n)$. However, for multiple outliers the proposed SMEB (4) would achieve improved robustness, compared with (6). For multiple outliers, the inaccurate estimator θ_n may cause $|e_n| \approx \nu\theta_n$. In this case, $\nu\theta_n/|e_n| \ll \nu\theta_n/(|e_n| - \nu\theta_n)$. Therefore, (4) achieves improved robustness with respect to multiple outliers. The improved robustness had been verified in the simulations.

3.2. Robust BEACON with time-varying set-membership low bound (robust BEACON-VB)

Here, we use an error bound function that can automatically adjust the set-membership low bound with the update of the filter coefficients [9]. The time-varying set-membership low bound is $\gamma_c = \eta_n$, which can be expressed as

$$\eta_n = \beta_2 \eta_{n-1} + (1 - \beta_2)\sqrt{\tau \|\hat{\mathbf{w}}_{n-1}\|^2 \times \sigma_\upsilon^2} \tag{7}$$

where $0 \ll \beta_2 < 1$ is a forgetting factor that should be adjusted to ensure an appropriate time-averaged estimate of the evolution of the power of the adaptive filter tap-weights $\hat{\mathbf{w}}_n$, τ is the tuning parameter, and σ_υ^2 is the variance of the impulsive-free noise. The tuning parameter in (7) should be set by experimental results and too high or low values will lead to inappropriate time-varying low bound. Table 1 shows a summary of the robust BEACON algorithm with time-varying set-membership low bound, which will be used for the simulations.

3.3. Discussion of computational complexity

The computational complexity of the robust BEACON-VB is compared with that of conventional RLS and BEACON-VB algorithms in terms of the total number of additions, multiplications,