Digital Signal Processing 46 (2015) 59–67

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

H_∞ filtering for networked linear systems with multiple packet dropouts and random delays

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ARTICLE INFO

Article history:
Available online 4 August 2015

Keywords:
H_∞ filtering
Packet dropout
Random delay
Networked system
Linear matrix inequality

ABSTRACT

This paper is concerned with an H_∞ filtering problem for the network-based linear systems with multiple packet dropouts and random delays. Due to the limited bandwidths of communication channels, the measured outputs will be delayed or even be lost during the transmission from the sensor to the remote filter, where the delays are randomly varying but bounded and packet dropouts are possibly consecutive. Based on a recent developed model that describes the phenomena of packet dropouts and time delays simultaneously, an H_∞ filter is designed to ensure the filtering error system to be mean-square exponentially stable and guarantee a prescribed H_∞ filtering performance. A sufficient condition for the existence of such a filter is provided via a linear matrix inequality (LMI) method. The effectiveness and applicability of the proposed algorithm is demonstrated by a practical F-404 aircraft engine system.

1. Introduction

With the rapid development of computer and communication technology, networks have been widely used as the medium in modern engineering systems to connect the spatially distributed sensors, actuators and controllers or filters. Such systems are the so-called networked control systems (NCSs) which have many advantages, such as low cost, simple installation and maintenance, convenient system diagnosis and increased system agility [1]. However, due to the limited bandwidths of the communication channels, the network-induced delays are inevitable during the data transmission through the networks from the sender to the receiver [2,3]. A transmission delay may be less or larger than one sampling period which is the so-called short time-delay or large time-delay. For example, the TCP/IP communication protocol has the large communication delays but the UDP/IP has generally the short communication delays [4]. The network-induced delays have the random characteristic in nature but are bounded. If the packet is with a delay longer than a certain pre-determined number, one possible strategy is to discard the packet and treat it as a packet dropout, which could deteriorate the system performance or even lead to the instability. So the random delays and packet dropouts are the two important issues which have attracted considerable research attention in the NCSs.

The filtering problem for networked systems has been a focus of research due to their important engineering applications such as target tracking, signal processing and control application [5–8]. It is well known that the Kalman filtering is the classical scheme to deal with the estimation problem effectively. In a network environment, the Kalman filtering should be modified to conduct the random phenomena [9–12]. However, one primary limitation of Kalman filtering is that the external disturbances are required to be Gaussian noises with known statistical property. Such a requirement is not always satisfied in practical applications. For this case, the H_∞ method is taken as an alternative method. A great number of important results for the H_∞ control problems have been reported [13–16]. As for H_∞ filtering problem, the objective is to minimize the highest energy gain of the estimation error for all initial conditions and noises where the noise signals are assumed to be arbitrary but with bounded energy or bounded average power rather than just Gaussian. Hence, the H_∞ filtering problem of networked systems has also received extensive research attention [17–22].

Due to the random nature of transmission delays and packet dropouts, they can be described by Bernoulli distributed white sequences or Markov chains [23–25]. As for the first type, a set of Bernoulli distributed random variables are used to establish the NCSs with multiple packet dropouts and random transmission delays simultaneously [26,27]. However, in [26], the model deals with the networks with retransmission mechanism which may cause the network congestion. A new model is established to describe the phenomena of multiple random delays and packet dropouts

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http://dx.doi.org/10.1016/j.dsp.2015.07.008

in [27] and the optimal linear filters with/without time stamps have been designed by the innovation analysis approach, which is generalized to multi-sensor systems with different delay and loss rates in [28] where a distributed fusion filter is designed. However, the exogenous disturbance of the system in [27,28] is required to be stochastic Gaussian noise with the known statistical information. When a priori information on the external noise is not available, the designed scheme is no longer applicable. In [29], the full-order H_∞ filter is designed based on the model of [27] to overcome the requirement of Gaussian noise. However, due to the effect of the multi-step time delay, the inequality scaling is used during the LMI transformation. So the obtained result is conservative. In [30], the H_∞ filtering problem is investigated by the delay partitioning method for the NCSs with the time-varying and bounded delay. But the lower bound of delay cannot be zero and the Lyapunov–Krasovskii functional candidate is complex. In recent literature [31], a new method is presented to deal with the bounded delay where the lower bound can be equal to zero. But the data packet dropouts are not considered.

In this paper, the H_∞ filtering problem is taken into account for discrete-time linear systems with random packet dropouts and time delays simultaneously. By introducing some new variables, the original system is modeled as a stochastic parameterized system and then the filtering error system is constructed. Sufficient conditions are proposed based on the Lyapunov stability analysis and LMI technique such that the derived filtering error system is mean-square exponentially stable with a prescribed disturbance attenuation level. An F-404 aircraft engine system is utilized to illustrate the effectiveness and applicability of the proposed algorithms. The main contributions of the paper are as follows: 1) The H_∞ filter is designed for discrete-time linear systems with multiple random delays and packet dropouts simultaneously. 2) Sufficient conditions are derived through Lyapunov stability analysis for the filtering error system to be mean-square exponentially stable and to achieve a prescribed H_∞ performance level. 3) A simple LMI approach is employed for solvability of the desired filter. Moreover, the conservatism can be reduced compared with the full-order filter design approach by inequality scaling during the LMI transformation. This makes structure of the filter more general for the NCSs with multiple packet dropouts and random delays.

The rest of this paper is organized as follows. Section 2 formulates the problem under consideration. The stability condition and H_∞ performance analysis of the filtering error system are given in Section 3. The filter design problem is solved in Section 4. An illustrative example is provided in Section 5 and the conclusions are drawn in Section 6.

Notation. Throughout the paper, I and 0 denote the identity matrix and zero matrix with suitable dimensions. $E\{\cdot\}$ stands for the mathematical expectation operator. The superscript T is the transpose operator. For a matrix $X \in \mathbb{R}^{n\times n}$, the notation $X > 0$ ($X < 0$) means X is real symmetric positive (negative) definite, and $\lambda_{\max}(X)$ or $(\lambda_{\min}(X))$ means the largest or smallest eigenvalue of X. Prob$\{\cdot\}$ denotes the occurrence probability of the event. $l_2[0,\infty)$ is the space of square summable vectors and I^+ is the set of positive integer. The symbol "$*$" in a matrix represents the symmetric term. diag$(\cdot)$ stands for a block-diagonal matrix.

2. Problem formulation

Consider the network-based discrete-time linear system:

$$\begin{cases} x_{k+1} = Ax_k + Bw_k \\ \tilde{y}_k = Cx_k + Dw_k \\ z_k = Lx_k \end{cases} \tag{1}$$

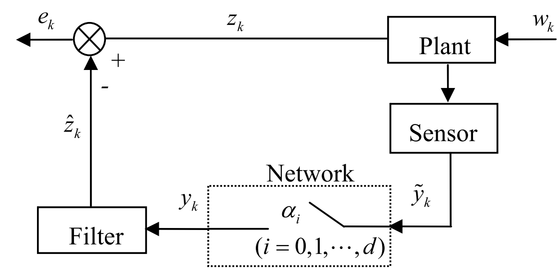

Fig. 1. Diagram of the networked system.

where $x_k \in \mathbb{R}^n$ is the state, $\tilde{y}_k \in \mathbb{R}^r$ is the measured output, $z_k \in \mathbb{R}^m$ is the signal to be estimated, $w_k \in \mathbb{R}^q$ is the exogenous disturbance input which belongs to $l_2[0,\infty)$, and A, B, C, D and L are known constant matrices with appropriate dimensions.

In the following discussion, we assume that the sensor and filter are clock-driven, and the sensor sampling, sending and the filter receiving are synchronous. When the data packets are transmitted through the networks from the sensor to the remote filter, the phenomena of transmission delays and packet dropouts unavoidably occur. Thus, the remote filter will receive the incomplete sensor information. The model to describe the network-induced issues is given by [27]

$$\begin{aligned} y_k = {} & \alpha_{0,k}\tilde{y}_k + (1-\alpha_{0,k})\Bigg\{(1-\alpha_{0,k-1})\alpha_{1,k}\tilde{y}_{k-1} \\ & + \big[1-(1-\alpha_{0,k-1})\alpha_{1,k}\big] \\ & \times \Bigg\{(1-\alpha_{0,k-2})(1-\alpha_{1,k-1})\alpha_{2,k}\tilde{y}_{k-2} \\ & + \cdots \left[1-\prod_{i=0}^{d-2}(1-\alpha_{i,k-d+i+1})\alpha_{d-1,k}\right] \\ & \times \prod_{i=0}^{d-1}(1-\alpha_{i,k-d+i})\alpha_{d,k}\tilde{y}_{k-d}\Bigg\}\Bigg\}\cdots\Bigg\}, \end{aligned} \tag{2}$$

where $y_k \in \mathbb{R}^r$ is the measurement received by the remote filter, $\alpha_{i,k}$ $(i = 0,1,\ldots,d)$ are Bernoulli distributed random variables which are uncorrelated with each other and satisfy the probabilities Prob$\{\alpha_{i,k} = 1\} = \bar{\alpha}_i$ and Prob$\{\alpha_{i,k} = 0\} = 1 - \bar{\alpha}_i$, where $0 \le \bar{\alpha}_i \le 1$, and d is the largest transmission delay from the sensor to the filter.

The structure of the considered networked system is illustrated in Fig. 1.

Remark 1. To avoid network congestion as possible, a packet at the sensor side is only sent once. The filter only receives one packet or nothing at each time, which can be met in the case of a single radio at the filter side.

Model (2) describes the possible one-step, two-step up to d-step transmission delays and packet dropouts where the delays are randomly varying and packet dropouts are possibly consecutive. Take $d = 2$ as an example, the measurement received by the filter can be given as follows:

$$\begin{aligned} y_k = {} & \alpha_{0,k}\tilde{y}_k + (1-\alpha_{0,k})\big\{(1-\alpha_{0,k-1})\alpha_{1,k}\tilde{y}_{k-1} \\ & + \big[1-(1-\alpha_{0,k-1})\alpha_{1,k}\big](1-\alpha_{0,k-2}) \\ & \times (1-\alpha_{1,k-1})\alpha_{2,k}\tilde{y}_{k-2}\big\}. \end{aligned}$$

The data transmission case for $d = 2$ is given in the following Table 1.