Digital Signal Processing 46 (2015) 133–150

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

An efficient algorithm for harmonic retrieval by combining blind source separation with wavelet packet decomposition

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ARTICLE INFO

Article history:
Available online 29 August 2015

Keywords:
Blind source separation (BSS)
Harmonic retrieval
Wavelet packet (WP)
Mutual information (MI)

ABSTRACT

In the present paper, we propose an efficient framework and algorithm for one dimensional harmonic retrieval problem in additive colored Gaussian or non-Gaussian noise when the frequencies of the harmonic signals are closely spaced in frequency domain. Our framework utilizes the wavelet packet (WP) method to the blind source separation (BSS) based harmonic retrieval model. Firstly, we establish the BSS based harmonic retrieval model in additive noise using only one mixed channel signal, at the same time, the fundamental principle of BSS based harmonics retrieval algorithm is analyzed in detail. Then, the harmonic retrieval algorithm is developed mainly using the WP decomposition approach, where the criterion is formed as the cumulant based approximation of the mutual information (MI) for the selection of optimal sub-bands of WP decomposition with the least-dependent components between the same nodes. Simulation results show that the proposed algorithm is able to retrieve the harmonic source signals and yield good performance.

1. Introduction

The one- and multi-dimensional harmonic retrieval (HR) problem arises in various areas of signal processing including communications, geophysics and radar signal processing, which has been an active research area during the past few decades [1–10]. HR in additive colored noise is one of several important research topics in which the number of harmonics and their frequencies often need to be estimated from noisy data accurately, especially when the frequencies of the harmonic signals are very close in frequency domain and corrupted with additive colored Gaussian or non-Gaussian noise.

The realization of HR in additive noise includes the traditional approach and modern methods. The traditional approach was achieved by the Fourier transformation (FT) method, which is stable but the resolution of the solution is low. In order to increase the resolution of the spectral estimation, some modern methods have been developed, such as maximum entropy, forward and backward least square (LS) spectral estimation method, singular value decomposition (SVD) based backward prediction method, forward and backward prediction method, SVD-total least square (SVD-TLS) method and modified Yule–Walker (YW) method [1–3], but all these methods have low signal-to-noise ratio (SNR).

In addition, most HR approaches either assumed white noise and utilized correlation based methods or assumed colored Gaussian noise and employed higher-order statistic based methods [2,3,8]. The former class methods based on second-order statistics generally assume that either the noise is white or its covariance matrix is given, which include Prony's method, several versions of the YW approach, and methods that are based on Pisarenko's procedure. Even though there has been some work addressing colored noise, typically, however, the noise is colored and *a priori* knowledge (or an estimate) of the noise covariance matrix is unavailable. By exploiting the fact that higher-than-second-order cumulants are zero for Gaussian processes, several researchers have demonstrated that cumulant-based HR methods can suppress the effect of colored Gaussian noise [2,8]. In all these methods, it is commonly assumed that the additive noise has Gaussian distribution or non-Gaussian distribution but the model of the noise must be restricted.

The resolution of harmonic signals can be described as follows. If $x(t)$ consists of two harmonics, at frequencies f_1 and f_2, there are two peaks in the power spectrum. The resolution of an estimator is the smallest value of $|f_1 - f_2|$ that leads to two discernible peaks. The resolution of the classical power spectrum estimators is of order $1/T$, where T is the effective window length. In contrast,

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http://dx.doi.org/10.1016/j.dsp.2015.07.010

the resolution of the periodogram is $1/Q$, where Q is the data length, and since $T \ll Q$, the periodogram exhibits higher resolution than the classical power spectrum estimators. But, the choice of the window function dictates the resolution-variance tradeoff: if the corresponding frequency domain representation of $x(t)$ has a broad main lobe, the power spectrum estimate will be smoother, the variance of the estimate will be smaller, and as a cost the resolution will be lower. When the frequencies of harmonic signals are closely spaced in the frequency domain, the problem is that we cannot get ideal identification results using the proposed conventional HR methods.

In this paper, we don't make assumptions about the distribution, color and model of the additive noise except that it is stationary.[1] The proposed HR algorithm is realized by using the wavelet packet (WP) decomposition approach and the commonly used method called blind source separation (BSS), in addition, we just utilize a single channel mixture of the harmonic source signals. Note that, in some sense, the proposed algorithm in this paper is an optional procedure of a particular case of instantaneous frequency (IF) estimation problem [11].

BSS is typically based on the assumption that the mixed signals are a linear superposition of underlying hidden source signals. When the source signals are mutually independent, the BSS can be solved by using the so called independent component analysis (ICA) method which has attracted considerable attention in the signal processing and neural network fields and several efficient algorithms have been proposed [12–14]. The basic task of BSS is estimating part of source signals that are linearly combined in mixtures which has received wide attention in various fields such as biomedical signal processing, speech and image recognition and enhancement, geophysical data processing, data mining, wireless communications and so on.

Linear under-determined blind source separation (UBSS) problem, where the number of the source signals is more than that of the mixed signals, is a common issue in practice especially in speech separation and biomedical signal processing [12–14]. In this case, the inverse of mixing matrix does not exist and, consequently, a solution for source estimation should also be found even if the mixing matrix has been estimated, which makes the conventional ICA based BSS algorithm cannot separate or extract all the potential source signals successfully. To dispose of linear UBSS problem, some *a priori* information should be resorted, a powerful framework for solving linear UBSS is to exploit the sparsity of source signals in a given signal representation dictionary [14].

In this paper, we first establish the BSS model of HR problem using the only one channel mixed signals. Under the multiresolution decomposition frame, using the WP based iterative filter banks, we propose an algorithm to the HR which is achieved using the established BSS model. The algorithm can adaptively select the nodes in the WP trees which have the least dependent components of the harmonic signals according to the criterion based on efficient approximation of the mutual information (MI), which is a consistent estimator of the MI and computationally more efficient than entropy based estimator [15].

This paper is organized as follows, Section 2 gives the BSS based HR model; Then in Section 3, we describe the separation conditions and novel WP based HR algorithm called WP-BSS-HR; Simulation experiments illustrating the performance of the proposed method are given in Section 4; Finally, Section 5 concludes the paper.

[1] Most HR retrieval algorithms paid much more attentions on the distributions or/and temporal structures of the noises in the HR model, the noises were under the "stationary" assumption [5,8–10]. So, in the manuscript, following this hypothesis, we assume that the noise is "stationary" and do not give any more assumptions on it.

2. Problem formulation

2.1. BSS model

Let us denote the P source signals by the vector $\mathbf{s}[k] = (s_1[k], s_2[k], \ldots, s_P[k])^{\mathrm{T}}$, and the mixed signals by $\mathbf{x}[k] = (x_1[k], x_2[k], \ldots, x_Q[k])^{\mathrm{T}}$, where k is the time index. Now the mixing process can be expressed as

$$\mathbf{x}[k] = \mathbf{A}\mathbf{s}[k] + \mathbf{n}[k] \tag{1}$$

where the matrix $\mathbf{A} = [a_{ij}] \in \mathbf{R}^{Q \times P}$ collects the mixing coefficients. No particular assumptions on the mixing coefficients are made. However, some weak structural assumptions are often made, for example, it is typically assumed that the mixing matrix is square, that is, the number of source signals equals the number of mixed signals ($P = Q$), the mixing process $\mathbf{A}$ is defined by an even-determined (i.e. square) matrix and, provided that it is non-singular, the underlying sources can be estimated by a linear transformation $\mathbf{W}$. If $Q > P$, the mixing process $\mathbf{A}$ is defined by an over-determined matrix and, provided that it is full column rank, the underlying sources can be estimated by least-squares optimization or linear transformation involving matrix pseudo-inversion. If $Q < P$, then the mixing process is defined by an under-determined matrix and consequently source estimation becomes more involved and is usually achieved by some non-linear techniques. For technical simplicity, we shall also assume that all the signals have zero mean, but this is no restriction since it simply means that the signals have been centered. $\mathbf{n}[k] = (n_1[k], n_2[k], \ldots, n_P[k])^{\mathrm{T}}$ is a vector of additive noise. The problem of BSS is now to estimate both the source signals $s_j[k]$, $j = 1, \ldots, P$ and the mixing matrix $\mathbf{A}$ based on observations of the $x_i[k]$, $i = 1, \ldots, P$ alone [12–14].

Recall that two indeterminacies cannot be resolved in BSS without some *a priori* knowledge: scaling and permutation ambiguities. Thus, if the estimate of the mixing matrix $\hat{\mathbf{A}}$ satisfies $\mathbf{B} = \mathbf{W}\mathbf{A} = \hat{\mathbf{A}}\mathbf{A} = \mathbf{G}\mathbf{D}$, where $\mathbf{B}$ is a global transformation matrix which combines the mixing and separating system, $\mathbf{G}$ is some permutation matrix and $\mathbf{D}$ is some nonsingular scaling diagonal matrix, then $(\hat{\mathbf{A}}, \hat{\mathbf{s}})$ and $(\mathbf{A}, \mathbf{s})$ are said to be related by a waveform-preserving relation.

To make the BSS problem simpler and the rate of convergence faster [12–14], a linear transformation $\mathbf{V}$ called pre-whitening is often used to transform the mixed signals $\mathbf{x}$ to $\tilde{\mathbf{x}} = \mathbf{V}\mathbf{x}$ such that

$$E\left[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^{\mathrm{T}}\right] = \mathbf{I} \tag{2}$$

where $\mathbf{V}$ is a pre-whitening matrix.

2.2. BSS based HR model

In general, the discrete noiseless one-dimensional real harmonic signals which contain P sinusoids are modeled as follows:

$$s[k] = \sum_{p=1}^{P} a_p \cos[\omega_p k + \varphi_p] \tag{3}$$

where P is the number of the harmonic signals, a_p and ω_p are the unknown constants called amplitudes and frequencies, and $\omega_i \neq \omega_j$ for $i \neq j$, amplitudes are assumed positive and $0 < \omega_p < \pi$, $p = 1, 2, \cdots, P$. Additionally, the phases φ_p are i.i.d. random variables uniformly distributed over $(-\pi, \pi]$. Due to the presence of noise, one observes a noise contaminated version of $s[k]$, namely

$$x[k] = s[k] + n[k] \tag{4}$$

where $n[k]$ is the additive noise which is stationary and statistical independent to harmonic signals. The problem of interest is to estimate harmonic number P and their frequencies ω_p using just the one channel noisy mixture $x[k], k = 1, 2, \ldots, T$.