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A harmony search algorithm for high-dimensional multimodal optimization problems

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ABSTRACT

Harmony search (HS) and its variants have been found successful applications, however with poor solution accuracy and convergence performance for high-dimensional (≥200) multimodal optimization problems. The reason is mainly huge search space and multiple local minima. To tackle the problem, we present a new HS algorithm called DIHS, which is based on Dynamic-Dimensionality-Reduction-Adjustment (DDRA) and dynamic fret width (*fw*) strategy. The former is for avoiding generating invalid solutions and the latter is to balance global exploration and local exploitation. Theoretical analysis on the DDRA strategy for success rate of update operation is given and influence of related parameters on solution accuracy is investigated. Our experiments include comparison on solution accuracy and CPU time with seven typical HS algorithms and four widely used evolutionary algorithms (SaDE, CoDE, CMAES and CLPSO) and statistical comparison by the Wilcoxon Signed-Rank Test with the seven HS algorithms and four evolutionary algorithms. The problems in experiments include twelve multimodal and four complex uni-modal functions with high-dimensionality.
Experimental results indicate that the proposed approach can provide significant improvement on solution accuracy with less CPU time in solving high-dimensional multimodal optimization problems, and the more dimensionality that the optimization problem is, the more benefits it provides.

1. Introduction

High-dimensional multimodal optimization problems are more and more often encountered in our real applications, especially due to the big data gained from and the complex problems to be solved in our real world. They are challenging in that the search space is very large due to the high dimensionality of the problem (e.g., >200), and the too large number of modals (i.e., too many local minima) among which only one is the globally optimal. In the case that problem is with more than 1000 dimensions and possibly infinite number of local minima, it is of great challenge on how to search for the globally optimal solution in an efficient time.

In recent years, swarm intelligent algorithm casts a population of individuals to perform an effective heuristic random search in parallel with mutual learning process to realize global optimization for an optimization problem. It has received much attention in comparative to conventional mathematical optimization algorithms in that it is not limited by requiring substantial gradient information and not sensitive to initialization [1].

As a typical swarm intelligent algorithm and characterized by simplicity, utilizing real-number encoding and fewer mathematical requirements and so forth, harmony search (HS) and its variants [2–15,51–54], mimicking the process of improvising a musical harmony, have been found to be potential in solving optimization problems. They have been applied to many fields of science and engineering successfully (e.g., pipe network design optimization problems [16], structural optimization problems [17,18], nurse rostering problems [19], economic load dispatch problems [20–22], PID controller optimization problems [23], location of wireless sensor networks [24], trajectory planning for robots [25], vehicle routing optimization problems [26,27], reliability problems [28], 0–1 knapsack problems [29], feature selection [30,31] and so on [32–46]).

To our limited knowledge, the present HS and its variants have not been found applications to the high-dimensional multimodal optimization problems (e.g., the dimensionality is larger than 200). Possibly this is due to either the so high dimensionality and/or the so many modals (local minima). In this situation, algorithm

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should be very powerful in exploration; otherwise, the region of the globally optimal solution cannot be positioned; and it should be also very powerful in exploitation when the region has been positioned such that it can be very stable in the region with a good property of convergence to the globally optimal solution.

For solving high-dimensional multimodal optimization problems, balancing exploration power and exploitation power is especially important. The exploration is to find new regions in search space [8], where population diversity plays an important role. The exploitation, which expects to obtain a high precision solution, means the computing power of the algorithm by using the information that has already been collected before. Therefore, exploration power is strongly required before locating into the region that contains the globally optimal solution. When the region has been found, the exploration power of algorithm should be degraded and the exploitation power should be enhanced. A gradual transfer from exploration to exploitation should be given for the search without a sharp cut.

In this study, we propose a new HS algorithm called DIHS, which employs a new Dynamic-Dimensionality-Reduction-Adjustment (DDRA) strategy and dynamic fret width (*fw*) strategy. The DIHS can achieve a good balance between exploration and exploitation for solving high-dimensional multimodal optimization problems.

The rest of this paper is organized in the following way: Section 2 introduces the standard HS algorithm. Take-one strategy for fast convergence to globally optimal solution is introduced and DIHS algorithm is proposed in Section 3. In Section 4, four parameters ($S_{\max}$, $S_{\min}$, HMS and fw_{mid}) are investigated, sixteen high-dimensional benchmark functions and computation results about them are discussed, the convergence and robustness on DIHS are analyzed and a portfolio optimization problem is also used to investigate the performance of DIHS. Finally, conclusions are drawn in Section 5.

2. Harmony search algorithm

The optimization problem to be solved is below:

$$\underset{X}{\text{Minimize}} \quad f(\boldsymbol{X}), \quad X = (x_1, x_2, \ldots, x_D) \in S$$
$$\textbf{S.t.} \qquad x_i \in [x_{L_i}, x_{U_i}], \quad i = 1, 2, \ldots, D$$

where $S \subseteq R^D$, $\boldsymbol{X_L} = (x_{L_1}, x_{L_2}, \ldots, x_{L_D})$ and $\boldsymbol{X_U} = (x_{U_1}, x_{U_2}, \ldots, x_{U_D})$ respectively are lower and upper bounds of the available search space, D is the dimensionality of the problem, x_i $(i = 1, 2, \ldots, D)$ is decision variable.

The implementation of standard HS algorithm for solving optimization problem is as follows:

Step 1. Initialization of optimization problem and algorithm parameters: The optimization problem and the control parameters of HS algorithm are specified. Parameters include HMS, HM considering rate (HMCR), pitch-adjusting rate (PAR), fret width (*fw*) (fret width is called formerly bandwidth: *bw*) and the termination criterion (i.e., the maximum function evaluation times: MaxFEs).

Step 2. Initializing the HM with a uniformly distributed random number in search space S. HM is a matrix of size HMS × D.

Step 3. Improvising a new harmony $\boldsymbol{X}^{new} = (x_1^{new}, x_2^{new}, \ldots, x_D^{new})$ based on the following three rules:

For each note x_i^{new} $(i = 1, 2, \ldots, D)$
  If $r_1 <$ HMCR, perform rule (a): Harmony memory consideration rule.
    If $r_2 <$ PAR, perform rule (b): Pitching adjustment rule.
  Else perform random consideration rule (c) with probability 1-HMCR.
End
  where r_1 and r_2 are uniformly distributed random number between 0 and 1.

Step 4. If the $\boldsymbol{X}^{new}$ is better than the worst harmony in the HM, judged in terms of the objective function value, X^{new} replaces the worst harmony in the HM.

Step 5. Checking the stopping criterion. If stopping criterion (MaxFEs) is meet, computation is terminated. Otherwise, Step 3 and Step 4 are repeated.

3. The proposed algorithm

Due to the high dimensionality and the multi-modality of the optimization problem, one needs to consider many problems in great detail such that the globally optimal solution can be reached with an efficient computation time: under the constraint that one cannot provide too large number of samples in HM, (1) in the initial search process, exploration power should be as large as possible such that one cannot lose the region of the globally optimal solution; (2) whenever the solutions in the HM are suboptimal in that they are close to the globally optimal solution, the search should be as fast as possible to reach the globally optimal solution, rather than destroyed by search strategy; (3) the search is a gradual process which requires a smooth transfer from initialization to convergence.

For that purpose, and considering that the dimension is too high and the modals are possibly too many, we propose several strategies for efficiently finding the globally optimal solution with the HS approach.

3.1. Take-one strategy for a fast convergence to globally optimal solution

We consider an extreme case of solutions in the HM, referred to as extreme HM. Assume that we have reached suboptimal solutions in the HM being

$$\text{HM} = \begin{bmatrix} \boldsymbol{X}^1 \\ \boldsymbol{X}^2 \\ \vdots \\ \boldsymbol{X}^{\text{HMS}} \end{bmatrix} = \begin{bmatrix} \mathbf{y_1^1} & x_2^* & \cdots & x_{\text{HMS}}^* & \cdots & x_D^* \\ x_1^* & \mathbf{y_2^2} & \cdots & x_{\text{HMS}}^* & \cdots & x_D^* \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^* & x_2^* & \cdots & \mathbf{y_{HMS}^{HMS}} & \cdots & x_D^* \end{bmatrix}$$

where $\boldsymbol{X}^* = (x_1^*, x_2^*, \ldots, x_D^*)$ is the globally optimal solution. We need adjust only one dimension such that the new solution is exactly the globally optimal solution.

We now consider two strategies to see which one is more probable (in probability) to reach to exactly the globally optimal solution directly, as the dimensionality D increases. The probability that the new solution X^{new} is exactly the globally optimal solution X^* is referred to as success rate here.

The strategies are take-one and take-all respectively. In the take-one search, the new solution is simply a solution in HM (referred to as the base solution) with an exception of its some dimension whose value takes the value of that dimension of any solutions in the HM; in the take-all search the new solution is generated with its each dimension taking the value of that dimension of any solutions in the HM. Detail of the two strategies is below.