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A sparsity-perspective to quadratic time–frequency distributions

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ABSTRACT

We examine nonstationary signals within the framework of compressive sensing and sparse reconstruction. Most of these signals, which arise in numerous applications, exhibit small relative occupancy in the time–frequency domain, casting them as sparse in a joint-variable representation. We present two general approaches to incorporate sparsity into time–frequency analysis, leading to what we refer to as sparsity-aware quadratic time–frequency distributions. Both approaches exploit time–frequency sparsity under full data and compressed observations. In the first approach, quadratic time–frequency distributions are derived based on optimal multi-task kernel design. In this case, sparseness in the time–frequency domain presents itself as a new design task, adding to the two fundamental tasks of auto-term preservation and cross-term suppression. In the second approach, sparse reconstruction is used, in lieu of the Fourier transform, to obtain quadratic time–frequency distributions from compressed measurements observed in the time domain or the joint-variable domain. It is shown that multiple measurement vector methods and block sparsity techniques play a fundamental role in improving signal local frequency representations. Examples of both approaches are provided. Analysis is supported by results based on simulated data, electromagnetic modeled data, and real Doppler and micro-Doppler data measurements of radar returns associated with human motions.

1. Introduction

This review article deals with signal time–frequency (TF) signature reconstruction from complete and incomplete data. Incomplete data include missing observations or random sampling, and can be due to fading channels, discarding noisy measurements, hardware simplification, sampling frequency limitations, logistical restrictions on data collections and storage, or a result of co-existence between wireless communication systems and systems performing active or passive sensing [1–3]. The article groups recent developments and potential future advances in sparse nonstationary signal analysis into two fundamental approaches, both exploiting signal sparseness over the joint-variable TF domain.

There are numerous applications where nonstationary signals are present at the transmitter, receiver, or both. This covers astronomical, biological and man-made signals and spans ubiquitous active and passive sensing modalities, including sonar, radar, and ultrasound [1,4–6]. Nonstationary signals, especially frequency modulated (FM) signals, are typically employed by smart jamming [7,8], and also characterize speech and electromyographic recordings [9,10]. In radar and sonar systems, the Doppler frequency is used to estimate the radial velocity of a target which can be a constant, linear, or nonlinear function of time. The rotation, vibration, and coning motion of a target or its parts may produce periodic Doppler modulations of the received signal, referred to as the micro-Doppler effect, which are best revealed in the time–frequency domain [11–17].

Unlike sinusoidal signals, FM signals are wideband, that is, the signals occupy the entire bandwidth under Nyquist sampling. In this respect, they are not globally sparse when represented in the frequency domain. However, owing to their power concentration in frequency at the different time instants, these signals are instantaneously narrowband. In this regard, the time–frequency signatures of a large class of nonstationary signals occupy small regions in the TF domain. This property casts the signals as sparse in the joint-variable representations [18] and has recently invited sparse signal reconstruction and CS techniques [19–23] to play an important role in TF signal analysis and processing [24–28]. Opposite to stationary signals, where frequency sparsity can be assumed globally, local frequency reconstruction of a single- or a multi-component nonstationary signal is deemed to outperform the signal global reconstructions in which all data measurements are considered.

The objectives of recent research activities in sparsity-aware quadratic time–frequency distributions (SA-QTFDs) is to 1) utilize

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the signal joint-variable domain sparsity, 2) combat the adverse effect of missing samples on time–frequency signal representations, and 3) define effective sampling and data collection strategies for signals with time-varying spectral characteristics. Meeting these objectives will lead to fast data acquisition, improved signal and target detection and classification in radar, communications, sonar, and satellite navigation. Further, the research in SA-QTFDs benefits other important emerging applications in the area of big data where sparsity of the joint-variable domain provides a vehicle for reduction in data storage as well as efficiency in data recovery.

QTFDs seek to combine the instantaneous power and spectral energy density of the signal into one TF domain representation. However, missing samples and randomly under-sampled nonstationary signals give rise to artifacts that spread over both the TF domain and the ambiguity domain, which are related by the two-dimensional Fourier transform [29,30]. These artifacts obscure the signal components and their instantaneous frequencies (IFs). Efforts and attempts to use traditional TF smoothing kernels for reducing missing samples artifacts proved ineffective and unsuccessful. This is because the ambiguity-domain low-pass filter characteristics, underlying signal-independent kernels which are applied to mitigate the signal cross-terms [31–36], offer limited benefits against an exceedingly noisy ambiguity function (AF). On the other hand, missing samples can misguide the signal-dependent adaptive kernels [37] into capturing the wrong areas of the AF. These difficulties and inabilities to apply QTFDs in their known nominal forms call for new approaches wherein sparsity can be either leveraged in TF kernel design or directly used in constrained optimization problems for TF signature reconstructions.

Compressed sensing (CS) has been studied extensively in many applications, including radar [38–43]. In CS, a sparse representation of a signal is projected onto a much smaller measurement space. This leads, in general, to decreasing the data-acquisition requirements from the time, logistic, and hardware complexity perspectives. It is then possible to record a small number of linear measurements of a signal and, from those measurements, reconstruct the complete set of all samples that can be recorded conventionally. The required number of observations is slightly higher than the signal sparsity level, but far fewer than the signal ambient dimension. Although vastly applied in many applications, little consideration has been given to sparse reconstruction of nonstationary signals.

We consider the problem of SA-QTFDs using complete and compressed observations, following two general approaches, both are important and key to the understanding of the offerings of sparsity in enhancing nonstationary signal analysis. In the first approach, new QTFDs, within Cohen's class, are introduced through a novel kernel design which has sparsity in the TF domain as one of its primary goals. These multi-task kernels combine low-pass filtering for reduced interference distributions (RIDs), with sparsity in the TF domain, yielding robustness to missing data. The result is a superior distribution over that obtained through conventional data-independent or data-dependent kernel design. Different sparsity measures applied in the TF domain can be used to solve for the optimum multi-task TF kernels.

In the second approach, one departs from Cohen's class and replaces the Fourier transform (FT), which connects time, time-lag, and ambiguity domains to the TF domain, by a corresponding linear dictionary and solves the respective sparse reconstruction and optimization problem. This approach underlines most of the recent contributions of SA-QTFDs. The work in [18,25,44,45] performs sparse reconstructions from windowed data in the time domain and, in this respect, it parallels the short-time Fourier transform (STFT) and the spectrogram. Sinusoidal and chirp atoms have both been used within each window to form the dictionary matrix with the latter outperforming the former due to its better approximation of the local frequency behavior of most FM signals. Aside from the time domain, sparse reconstructions are carried out in [24] from compressed observations in the ambiguity domain, and in [29] from the instantaneous autocorrelation function (IAF) domain, i.e., the time-lag domain. The difference between the ambiguity domain and the IAF domain is that missing samples in time lead to missing samples in the IAF, but not in the AF, which only becomes noisy. Additionally, reconstruction from the IAF domain allows the use of the dictionary matrix with a reduced size and enables exploitation of local sparsity over a short time period. A clear and fundamental role of a multiple measurement vector (MMV) model, block sparsity, and multi-task Bayesian compressive sensing (BCS) techniques in revealing the signal local power behavior was established in [46]. The MMV model can arise from using multiple data windows [47] reminiscent of the multiple window spectrogram [48–55]. BCS enables, through the priors, the incorporation of the contiguity property of most TF signatures and thus enhances sparse optimization solutions. It should be noted that, in the above two approaches, we deal with complex data whereas a missing sample implies that both the sample's real and imaginary parts are unavailable. Further, missing samples are drawn from a uniform distribution. As such, data with contiguous missing samples are unlikely to occur neither are they enforced. Reference [56] addresses this case and presents solutions based on empirical mode decomposition.

A hybrid approach combining the aforementioned two approaches can also be used. In this case, sparsity-aware TF kernels are designed and applied to the AF similar to the first approach. A sparsity measure is then used to produce a signal power distribution in the TF domain. In this respect, sparsity is used twice, in kernel design as well as in obtaining QTFDs from the ambiguity domain.

The above two approaches and their hybrid schemes constitute a nonparametric perspective to SA-QTFDs. There is also a parametric dictionary based approach which is directly applied to the time-domain data to estimate the signal parameters [27]. This parametric perspective is justified by the need, in many applications, to perform classification based on features related to the estimated signal parameters. However, this approach specifically deals with nonstationary signals with *a priori* known structures, such as chirps and sinusoidal FM signals. As such, it works well when there is a good match between the assumed and the actual signal characteristics, but remains sensitive to deviations from the assumed model.

This paper also considers interpolation as a method to deal with missing data and as an alternative to the above CS approaches. Upon obtaining the interpolated data, one can then proceed with the computation of TF distributions. Since missing data samples in time introduce missing samples in the time-lag domain, interpolation can be performed in either the time or the time-lag domain. It is shown in [57] that interpolation of the IAF outperforms data interpolation in time, as it acts on reducing cross-terms through its underlying low-pass filter characteristics. As such, it provides a better approximation of the TF signature when the complete data is considered. In this respect, IAF interpolation is better suited for QTFDs even though it exhibits more missing samples than those originally occurring in time. We compare QTFDs with and without data interpolation and contrast their performance with sparse signal reconstruction.

In addition to deterministic signals, QTFDs were used in the past to estimate the time-varying spectrum of nonstationary random processes [58–60]. When dealing with underspread nonstationary random processes, compressive sensing techniques have been recently applied to estimate the minimum variance spectrum [61]. This work parallels that in [24] for deterministic signals, but views the applied kernel and compressed observations in