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A new optimized Stockwell transform applied on synthetic and real non-stationary signals

Ali Moukadem *, Zied Bouguila, Djaffar Ould Abdeslam, Alain Dieterlen

University of Haute Alsace, MIPS Laboratory, Mulhouse, France

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ABSTRACT

The aim of this paper is to improve the energy concentration of the Stockwell transform (S-transform) in the time–frequency domain. A modified S-transform is proposed with several parameters to control the width of a hybrid Gaussian window. A constrained optimization problem is proposed based on an energy concentration measure as objective function and inequalities constraints to define the bounds of the Gaussian window. An active-set algorithm is applied to resolve the optimization problem. The optimization of the energy concentration in the time–frequency plane can lead to more reliable applications for non-stationary signals. The simulation results show a significant improvement of the proposed methodology most notably in the presence of noise comparing with the standard S-transform and existing modified S-transform in the literature. Moreover, comparison with other known time–frequency transforms such as Short-time Fourier transform (STFT) and smoothed-pseudo Wigner–Ville distribution (SPWVD) is also performed and discussed. The proposed S-transform is tested also on real non-stationary signals through an example of split detection in heart sounds.

1. Introduction

Time–Frequency analysis is a powerful tool to describe signals both in the time and in the frequency domain. It transforms a one dimensional signal $x(t)$ into a two-dimensional function of time and frequency $T_x(t, f)$ [1]. This can be done by several approaches. The first class of solutions is named the linear time–frequency representations methods which the well-known Short-Time Fourier Transform (STFT) and the Wavelet-Transform (WT) are part of the main concept used in those approaches lies in the signal decomposition into elementary parts (atoms) and tries to localize each part in time and frequency properly and simultaneously. The second approach concerning the Time–Frequency (TF) methods is the quadratic transforms which aim at distributing the energy of the signal over the two description variables: time and frequency. Each approach has some advantages and drawbacks; while linear time–frequency representations are intuitive they suffer from poor TF resolution in many cases. This depends on the windows used to analyze the signal. On the other hand, the quadratic transforms (the Wigner–Ville for example) have a high TF resolution. However, they suffer from cross-terms in multicomponent signals and may also suffer from inner interference for non-linear mono-component signal. There is no time–frequency method which can be considered as optimal for all applications.

The Stockwell transform (S-transform) can be considered as a hybrid between the Short Time Frequency Transform (STFT) and the wavelet transform [2]. It can be viewed as a frequency dependent STFT or a phase corrected wavelet transform. It has gained popularity in the signal processing community because of its easy interpretation and fast computation [3]. The S-transform has been shown high performance in classification and feature extraction problems applied on non-stationary signals, such as heart sounds [4–6], power quality signals [7], EEG signals [8] etc. Generally the S-transform uses a Gaussian window, whose standard deviation varies over frequency. Whatever the analyzed signal, the width of the Gaussian window will decrease as the frequency increases. This produces a higher frequency resolution at lower frequencies and a higher time resolution at higher frequencies. This can be considered as limitation in some signal analysis, for example, for a signal containing a single sinusoid, the time–frequency localization can be considerably improved if the window is very narrow in the frequency domain. Similarly, for signals containing only a Dirac impulse, it would be beneficial for good time–frequency localization to have very wide window in the frequency domain [9]. It would be more appropriate to adapt the window to the signal in order to maximize the energy localization of the S-transform.

Many studies in the literature tried to improve the Stockwell transform by proposing new windows. McFadden et al. [10] pro-

* Corresponding author.
E-mail address: ali.moukadem@uha.fr (A. Moukadem).

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posed a generalized S-transform which provides greater control on the window function. Later, Pinnegar et al. [11] proposed another generalized S-transform in which two prescribed functions of frequency control the scale and the shape of the analyzing window. The same authors proposed another modified S-transform with a bi-Gaussian window which seems better at resolving the sharp onset of events in a time series [12]. Sejdic et al. and Djurovic et al. [9,13] introduced a novel parameter to the Gaussian window and the parameter which maximizes the concentration energy is selected [9]. This is the main study in the literature interested to optimize the energy concentration directly in the TF domain, that is, to minimize the spread of the energy beyond the actual signal components. More recently, Assous et al. [14] proposed another modified S-transform in which the scaling parameter of the Gaussian window varies linearly with the frequency.

The energy concentration in the Time–Frequency (TF) domain is a very important criteria for the algorithms that aim to detect or extract relevant feature from time–frequency domain. Hence, the importance of an energy concentration optimization process to improve the detection and the classification of non-stationary signals. As it is well known, the ideal time–frequency transformation should only be distributed along frequencies for the duration of signal components. So the neighboring frequencies would not contain any energy and the energy contribution of each component would not exceed its duration [15]. In this paper, we adopt the strategy proposed by Sejdic et al. [9], that is, to adapt the analyzed window to the energy concentration criteria.

The main contributions of this paper can be summarized as:

- Proposing a methodology to optimize the energy concentration of the S-transform. For that, new parameters are introduced to control better the width of the Gaussian window and an active-set algorithm is applied to select properly these parameters.
- An application of the modified S-transform on the detection of splits in heart sounds is proposed.

This paper is an improved and extended version of the paper published in [16]. The paper is organized as follows: Section 2 presents the proposed modified S-transform with the optimization problem. Section 3 presents the simulation study to compare the proposed method to other existing S-transform where the robustness against noise and the performance of estimation of instantaneous frequency are discussed and detailed. A comparison with other classic time–frequency representations is also discussed. Section 4 tries to show the importance of the time–frequency resolution enhancement in the detection of real non-stationary signals by presenting an application on heart sounds. Finally, Section 5 gives the conclusion and the future work.

2. Optimization of the modified Stockwell transform

2.1. The original S-transform and the link with Fourier

The original S-transform of a time varying signal $x(t)$ is defined by [2]:

$$S_x(\tau, f) = \int_{-\infty}^{+\infty} x(t) w(t-\tau, f) e^{-2\pi j f t} dt \tag{1}$$

where the window function $w(t, f)$ is chosen as:

$$w(t, f) = \frac{1}{\sigma(f)\sqrt{2\pi}} e^{\frac{-t^2}{2\sigma(f)^2}} \tag{2}$$

and $\sigma(f)$ is a function of frequency as:

$$\sigma(f) = \frac{1}{|f|} \tag{3}$$

The window is normalized as:

$$\int_{-\infty}^{+\infty} w(t, f) dt = 1 \tag{4}$$

This gives the direct relation between the S-transform and the Fourier spectrum by averaging the local spectrum over time:

$$\int_{-\infty}^{+\infty} S_x(\tau, f) dt = X(f) \tag{5}$$

where $X(f)$ is the Fourier transform of $x(t)$. The signal $x(t)$ can be recovered from $S_\tau(t, f)$ as follows:

$$x(t) = \int_{-\infty}^{+\infty} \left\{ \int_{-\infty}^{+\infty} S_x(\tau, f) \right\} e^{i2\pi f t} df d\tau \tag{6}$$

Another way to directly express the link between the S-transform and the Fourier transform is by writing the ST as a convolutions process as follows:

$$\begin{aligned} S_x(\tau, f) &= \int_{-\infty}^{+\infty} p(t, f)\, g(\tau - t, f)\, dt \\ &= p(\tau, f) * g(\tau, f) \end{aligned} \tag{7}$$

where:

$$p(\tau, f) = x(\tau)\, e^{-i2\pi f \tau} \tag{8}$$

and:

$$g(\tau, f) = \frac{|f|}{\sqrt{2\pi}} e^{\frac{-\tau^2 f^2}{2}} \tag{9}$$

By calculating the Fourier transform of $S_x(\tau, f)$, the convolution becomes a multiplication in the frequency domain:

$$\begin{aligned} F_{\tau\to\alpha}\{S_x(\tau, f)\} &= P(\alpha, f)\, G(\alpha, f) \\ &= X(\alpha + f)\, e^{\frac{-2\pi^2\alpha^2}{f^2}} \end{aligned} \tag{10}$$

where $P(\alpha, f)$ and $G(\alpha, f)$ are as the corresponding Fourier transforms for $p(\tau, f)$ and $g(\tau, f)$, respectively and α is the frequency Fourier variable related to τ. The direct relation between the S-transform and the Fourier transform can be obtained by applying the inverse Fourier transform to the last equation:

$$S(\tau, f) = \int_{-\infty}^{+\infty} X(\alpha + f) e^{\frac{-2\pi^2\alpha^2}{f^2}} e^{i2\pi\alpha\tau} d\alpha \tag{11}$$

This will facilitate the implementation of the ST by using the advantages of the FFT (Fast Fourier Transform) algorithms. The exponential function in Eq. (10) is the frequency dependent localizing window. This window is centered on the zero frequency and thus plays the role of a low pass filter for each particular voice.

2.2. Energy concentration enhancement

It has been shown that the original S-transform uses a Gaussian window, whose standard deviation varies over frequency. Whatever the analyzed signal, the width of the Gaussian window will decrease as the frequency increases. As we mentioned it above, this strategy can be considered as a limitation since it does not