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Precision spectral peak frequency measurement using a window leakage ratio function

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ABSTRACT

For power spectra of signals consisting of stationary sinusoids mixed with random noise, the frequency and amplitude of a spectral peak can be estimated with greater accuracy than the nearest frequency bin of the Fourier transform by exploiting the spectral leakage characteristics for the particular data window used. Techniques such as linear interpolation or an amplitude weighted average have inadequate precision due to the nonlinear leakage into adjacent bins and the dependence on data window type. This paper offers a new general algorithm presented using the Fourier coefficients c_k of the input data window to produce a function which is the ratio of the side-bin amplitudes of the window in the frequency domain. The ratio function allows one to use the amplitudes of the adjacent bins of a spectral peak to precisely estimate the peak frequency and amplitude when the frequency does not lie exactly on a frequency bin (in between the discrete bins of a Fourier transform). Examples are provided for a number of popular data windows. The ratio function can be most easily implemented using a simplified log-ratio function for the window side bin magnitudes. A statistical analysis provides a useful frequency estimation error estimate given the signal-to-noise ratio of the spectral peak based on an approximation of the ratio of non-zero mean Gaussian variables. The benefits of this technique are not just improved estimation accuracy for amplitude and frequency, but also allow large spectral data files to be accurately reduced in size for remote monitoring of vibration spectra. An example is given of a methodology for reduction of spectral data file size without the loss of important signals for analysis where the file size is reduced by 88% with only a few percent error, which is mostly confined to the background noise in the reconstructed spectrum.

1. Introduction

Frequency measurement using discrete Fourier series on finite length sequences has long been a topic of interest in signal processing [1]. The use of data windows to control spectral leakage reduces frequency resolution but offers many other benefits such as producing more consistent spectral amplitudes when the actual frequency does not align precisely with the discrete spectral bin frequency in the Fourier transform. This led to the development of a wide range of data window types with various tradeoffs between spectral leakage and frequency measurement accuracy [2]. If higher frequency measurement resolution is needed one could either change the data window or use a longer Fourier transform

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input sequence and larger-sized Fourier transform. If the frequency of interest is known one could synchronize the analog to digital sampling rate so that the sinusoid and all of the harmonics lie exactly on discrete spectral bins in the Fourier transform [3]. In communications, precise estimation of a carrier frequency has been accomplished using a warped Fourier transform [4]. Recently, a method has been developed to resample the input for precise order tracking of harmonics [5]. Frequency estimation precision is very important in the area of rotating machinery analysis where vibration harmonics are to be associated with specific machinery components.

Modal frequency analysis resolution in structural transfer functions is driven by the structural damping rather than the Fourier transform limitations [6]. For transfer functions, the precision of the magnitude and phase is degraded by interfering noise or because the reverberation decay of a given structural resonance is longer than the size of the Fourier transform input sequence. Techniques have been developed for analysis of the transfer function errors due to spectral leakage and signal buffer overlap averaging [7].

In this paper we focus on the precision measurement of frequencies from stationary sources where the frequency is not aligned with a discrete spectral bin in the Fourier transform. The technique of amplitude-weighted interpolation is known to only be accurate when the true frequency is very close to a discrete spectral bin and the interpolation error depends on the type of data window used. Since the data window and its frequency response are known, we develop a technique here to exploit the window leakage as a means to estimate the precise frequency of the spectral peak. Since the ratio of the nearest two side bin amplitudes of a spectral peak will be unity when the actual frequency is perfectly aligned with a discrete spectral bin and irrational when the actual frequency is not aligned, we use a ratio function of the side bin leakage to estimate the actual frequency location between the discrete spectral bins of the Fourier transform. The known spectral leakage response also allows a small amplitude correction for the actual frequency peak of interest.

Besides offering an improved frequency and amplitude estimation accuracy, our technique also offers an accurate data reduction strategy for reducing the file size of vibration spectra since the known window response and peak amplitude and fractional frequency bin estimate allow many spectral bins to be accurately represented by these two numbers. For most rotating machinery vibration spectra this can result in a reduction in file size of more than 90%, which is important when considering equipment monitored via telemetry and data storage costs.

2. Spectral window responses in the frequency domain

We begin by considering an N-point discrete Fourier transform with no data window applied (this is the rectangular window case).

$$D_N(\Omega_b) = \sum_{n=0}^{N-1} e^{-j\Omega_b n} \tag{1}$$

The digital frequency is denoted as $\Omega_b = \frac{2\pi f_b}{f_s}$ and f_b is the frequency of the bth bin in Hz and f_s is the sampling frequency in samples per s. Therefore, $b = \frac{\Omega_b N}{2\pi}$ and for the commonly used radix-2 fast Fourier transform (FFT) the spectrum is evaluated over N equally spaced frequency bins where N is an integer power of 2. For a discrete Fourier transform (DFT), any frequency, or number of frequencies can be evaluated in the spectrum, but the resolution of the spectrum is by the size of by N and the type of data window used (unity for the rectangular window case). Using the formula for the geometric sum of a finite series

$$\sum_{k=0}^{N-1} ar^k = a\frac{1-r^N}{1-r} \tag{2}$$

Eq. (1) is reduced to the familiar form

$$D_N(\Omega_b) = e^{-j\Omega_b(N-1/2)} \frac{\sin(\Omega_b N/2)}{\sin(\Omega_b/2)} \tag{3}$$

or, in terms of the spectral bin number

$$D_N(b) = e^{-j2\pi b/N(N-1/2)} \frac{\sin(\pi b)}{\sin(\pi b/N)}. \tag{4}$$

Eq. (3) can be seen as a Dirichlet kernel [8] [9] which approximates a periodic Dirac delta function as $N \to \infty$ with period N in the frequency domain. The significance of the Dirichlet kernel for a DFT is that we can convolve it with the data window to define the frequency resolution and leakage response for a sinusoid in the windowed DFT. Further considering that $N \gg 1$

$$D_N(b) \approx e^{-j\pi b} \frac{N \sin(\pi b)}{\pi b} \tag{5}$$

which is the well-known result and is approximately N at $b=0$. Negative frequency bins are repeated in the $N-b$ bin between the $N/2$ and the $N-1$ DFT bin assuming a real input signal for the DFT. Consider the Hanning window [2] (named