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Phase difference methods based on asymmetric windows

Jiufei Luo [a,*], Ming Xie [b]

[a] School of Mechanical Engineering, Chongqing University, Chongqing 400044, PR China
[b] School of Automotive Engineering, Chongqing University of Technology, Chongqing 400054, PR China

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ABSTRACT

A new phase difference correction method, based on asymmetric windows, is proposed to correct the errors of frequency, phase, and amplitude in discrete spectra. Classic windows show the constant time delay and the same linear phase response due to their symmetric properties. Removal of the symmetry constraint on windows can result in variable time delays and different spectral phase responses. Based on the phenomenon of alterable spectral phases in asymmetric windows, we present a new approach, which could overcome inherent defects in traditional correction methods. The simulation results show that the new method can correct the errors of neighboring frequency components with high precision through the selection of proper asymmetric windows. Moreover, the new method can avoid the errors induced by a mistaken location of the spectral line, which is one of the major problems in traditional correction methods. When the symmetric windows in traditional methods are replaced by asymmetric windows, the improved algorithm also exhibits stronger robustness against additive noise. The comparative study reveals that each algorithm has its own advantages. In general, when the SNR is high, the interpolated method is a good choice for its simplicity; while the phase difference based methods provide better performance when the SNR is low. The new approach is recommended for closely spaced components or when high noise is involved.

1. Introduction

In discrete spectrum analysis, the picket-fence effect and the leakage effect are commonly unavoidable due to the limited duration observation interval and the difficulty of integral period truncation when sampling. The effects will lead to errors in frequency, amplitude, and phase even in the absence of noise. The technique of windowing can alleviate the pitfalls to some extent, but it cannot eradicate them. A well-known example cited by many researchers is that the maximum amplitude error amounts to respectively 36.4% and 15.3% for the analysis of a single harmonic with a rectangular window and a Hanning window [1,2]. The maximum frequency error is about a half of the frequency resolution for any kind of window [2]. The maximum error in phase is even up to 90° [3,4]. Therefore, the solution to the errors in the discrete spectrum would be of great significance for practical engineering applications. As a result, plenty of methods have been proposed to correct the errors in recent decades. In general, these methods can be divided into four groups: the method based on interpolation, the method based on FT continuous zoom, the method based on spectrum energy, and the method based on phase difference [3].

* Corresponding author. Tel.: +86 23 6511 2509.
E-mail address: jiufluo@gmail.com (J. Luo).

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Since the 1970s, the interpolation-based correction method [5] has been extensively studied and widely applied in various fields [1,6–18]. The essence of this method is to obtain the frequency estimate of a signal with the weighted average of a certain number of known spectral bins. Schoukens et al. analyzed the systematic errors and the noise sensitivities of different interpolation algorithms in a qualitative way [10]. Offelli and Petri analyzed the signals corrupted by additive white noise and deduced equations to display the way that the selection of windows, the number of samples and the signal-to-noise ratio (SNR) affect the interpolation method [11]. The method based on FT continuous zoom, also called the search method, obtains the frequency by continuously zooming in on the frequency segment concerned with [19,20]. The DFT is repeatedly carried out to get the highest spectral line whose frequency is the right value to be solved. It is easy to understand and implement the algorithm, but the calculation of the algorithm is time-consuming [21]. The energy-based method was first proposed by Offelli and Petri [22]. Ding and Jiang were the one who proved the theorem that the barycenter of the power spectrum of symmetric window functions is the coordinate origin [23]. This means that the energy-based method is suitable for any window. The interferences from spectral components, wideband noise, and other precision factors related to the estimated parameters have been comprehensively investigated [24–27]. Recently, this approach was also compared with other approaches [26].

The fourth algorithm is the phase difference correction method. This method was initially proposed in 1986, by McMahon and Barrett, as phase interpolation estimator (PIE) to estimate the frequency of a single tone with noise [28]. Then, it was further improved and applied in frequency estimation by Zhu et al. [29] and Santamria et al. [30]. In 1999, Xie and Ding [3,31] found a special case of the time-shifting correction method of the phase difference by using two continuous time-domain signal segments. In 2002, the universal phase difference correction method based on time shifting was derived by Ding [32]. In 2003, Huang [33] proposed another new algorithm based on the translation of the window center. At the same time, Ding and Xie [34] put forward the synthesis method, which involves time-domain translation and the change of window width. From a different point of view, a similar idea called the phase regression approach was proposed [35–36]. In 1989, Lang and Musicus provided an explanation for the threshold behavior of phase difference methods [37]. In 2002, the special case of the method based on time shifting in the presence of noise was studied in depth by Zhu [38]. Studies also revealed that the phase difference based methods show higher resistance in a noisy condition [21].

However, the traditional phase difference correction methods are established on symmetric windows. From the knowledge of authors, the asymmetric windows have not been deployed in a spectral correction method. This paper aims to introduce the asymmetric windows into the field of phase difference correction methods. First, a new phase difference method based on asymmetric windows was introduced to correct the errors of frequency, amplitude, and phase in discrete spectrum. Then, the performance of the new algorithm was compared with those of the traditional algorithms. This provides a theoretical basis for selecting an appropriate estimation method in practice.

The paper is organized as follows. In Section 2, the phase and magnitude characteristics of window functions (both symmetric and asymmetric functions) are explored to provide the theoretical basis for a correction method based on asymmetric windows; in Section 3, the basic theories of the traditional methods and the new method are briefly described; in Section 4, the effectiveness of these methods are verified through computer simulations, the new algorithm is compared with the commonly used algorithms, and the performance of those algorithms in the case of a cosine wave corrupted by additive white noise is studied, while the problem of the mistaken location of the spectral line is also analyzed; finally, the main conclusions are summarized in Section 5.

2. Characteristics of asymmetric windows

Windows are weighting functions which are introduced to reduce the spectral leakage caused by limited observation intervals. In 1978, Harris [39] issued his celebrated paper, in which a plethora of windows were discussed and analyzed. In 1981, Nuttall [40] presented the corrected plots of Harris' windows and also deduced additional windows with good side-lobes and optimal behaviors under different constraints. Windows have been extensively studied and widely applied in different fields of signal processing.

All classic windows are symmetric. Symmetric windows are characterized by the simple design and the property of their linear phase. While, this symmetric property leads to potential drawbacks, like constant time delays and frequency response limitations [41]. Relaxing the constraint of symmetry can, therefore, create asymmetric windows for special uses [41–44]. Due to the property of symmetry and the causality-imposed time shift, classic windows have the same spectral phase [42]. Consequently, the indicators for comparing different classic windows are established on the consideration of their magnitude response [39]. The asymmetric window functions, however, exhibit quite different spectral phase responses.

Before further discussion, we first introduce an establishment method for asymmetric windows. A simple way to obtain an asymmetric window is to truncate a function within a symmetric window. In other words, an asymmetric window $\hat{w}(t)$ is generated by multiplying a time-shifted symmetric window $w_T(t)$ by a function $s(t)$ which provides asymmetry as $\hat{w}(t)=w_T(t)s(t)$. A good example of the asymmetric window is obtained by truncating a straight line in different symmetric windows. The general expression form of a straight line passing through the origin is $s(t)=\rho t$, where ρ denotes the slope. Because of the linear property of the Fourier Transform, the parameter ρ has no effect neither on the magnitude shape nor on the phase. As a result, we may simply rewrite it as $s(t)=t$. It is worthwhile to note that we can adjust the scale factor ρ to make the maximum value of the asymmetric window equal to one (like the symmetric windows) or to get the normalized log-magnitude response. As is shown in Fig. 1, this window is constituted using the Hanning window and the straight line