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A two-stage fast Bayesian spectral density approach for ambient modal analysis. Part I: Posterior most probable value and uncertainty

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ABSTRACT

A two-stage fast Bayesian spectral density approach is proposed in this study to extract modal properties for the cases of separated modes and closely spaced modes. A novel technique for variable separation is developed so that the interaction between spectrum variables (i.e., frequency, damping ratio as well as the spectral density of modal excitation and prediction error) and spatial variables (i.e., mode shape components) can be decoupled completely for both cases. In a first stage, the spectrum variables can be identified through a so-called 'fast Bayesian spectral trace approach' (FBSTA) by employing the statistical properties of the sum of auto-spectral density, while the spatial variables can be estimated in a follow up second stage through 'fast Bayesian spectral density approach' (FBSDA) by using the statistical information of the entire spectral density matrix. This study also reveals the intrinsic relationship between FBSDA and 'fast Bayesian FFT approach' formulated recently when multiple sets of measurements are available. The newly developed two-stage Bayesian approach allows for a fast computation of the most probable values and covariance matrix of modal properties. The companion paper is devoted to assembling the local mode shape components corresponding to different setups to form the overall mode shapes using a Bayesian statistical framework and verifying the proposed algorithms through simulated and field testing data.

1. Introduction

Modal analysis has widespread applications in the fields of structural vibration control, structural health monitoring and structural damage detection. Primarily, modal properties include the natural frequencies, damping ratios and mode shapes. Remarkable progress has been made on experimental modal analysis based on utilizing both input and output measurement data. These approaches, however, are not readily applicable in large-scale civil infrastructure applications since they usually require special experiments which are often time consuming, obtrusive, and costly [1]. Therefore, ambient modal analysis using output-only measured response has aroused increasing interest in industrial applications since it can be carried out in a much more economical and efficient manner. An increasing number of ambient modal analysis approaches in the frequency domain have been developed during the past decades. Among others, the peak-picking method [2], frequency-domain decomposition (FDD)

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method [3], PolyMAX method [4], transmissibility based operational modal analysis approach [5,6], etc., are techniques capable of extracting the modal properties quickly. Attention has also been devoted to identifying the dynamic properties in the time domain. Well-known approaches include the eigensystem realization algorithm (ERA) [7] with data correlation, the stochastic subspace identification (SSI) method [8], etc.

Recent interest has arisen to calculate the uncertainties of modal parameters by using Bayesian approaches [9,10] which views modal analysis as an inference problem with probability interpreted as a measure for the relative plausibility of outcomes given a model of the system and measured data [11]. In the recent decade, Bayesian methods have a wide range of applications in dynamic analysis of civil and mechanical engineering, etc. Specially, there has been substantial development in Bayesian approach for structural system identification [9,12,13]. The issue of identifiability of the model parameters has also been extensively discussed by Katafygiotis et al. [14–16]. The comprehensive historic development of Bayesian methodologies as well as their applicability has been provided by Yuen et al. [17] and Beck [18].

In the context of ambient modal analysis, a number of Bayesian approaches including the Bayesian spectral density approach (BSDA) [19], Bayesian time domain approach (BTDA) [20,21], and Bayesian FFT approach (BFFTA) [22] have been proposed. Compared to BTDA, BSDA and BFFTA working in the frequency domain are more promising candidates for ambient modal analysis since they can utilize data in selected resonance bandwidths so as to legitimately avoid using information from across the entire frequency band [23]. These methods provide rigorous means for obtaining modal properties as well as their uncertainties for given measured data and modeling assumptions. However, computational difficulty has severely hindered their wider applications even for a moderate number of measured dofs. The computational challenges of the conventional Bayesian FFT approach were to a large extent addressed through a breakthrough contribution recently made by Au [23–25]. According to this, the most probable values as well as the posterior covariance matrix can be computed rapidly, allowing these results to be obtained even on site. The relationship between the Bayesian method and a frequentist approach in system identification were also investigated in detail [26]. Field applications in different engineering structures, such as a coupled floor slab system [27], a primary-secondary structure [28], and a super-tall building under strong wind [29], etc., have demonstrated the efficiency of these approaches.

Motivated by the fast Bayesian FFT approach [23–25], a two-stage fast Bayesian spectral density approach is proposed for ambient modal analysis in this paper so as to address the difficulties of conventional BSDA [19]. The approach is presented in two companion papers. This paper (Part I) presents the theory of identifying the most probable values and estimating the posterior covariance matrix. Following the tactic of ‘divide and conquer’, one can divide the spectral density bandwidth into a series of frequency sub-bands, and each selected resonant frequency sub-band can be conquered via a two-stage approach. The spectrum variables independent of spatial information, including natural frequencies, damping ratios as well as the magnitude of modal excitation and prediction error, can be separated in the first stage from the full set of modal parameters, and can be identified through a ‘fast Bayesian spectral trace approach’ (FBSTA) using the sum of auto-spectral densities of all measured dofs. The posterior covariance matrix can also be computed efficiently from analytical expressions of the Hessian matrix. Then, in the second stage, the mode shapes and their uncertainties are extracted instantaneously through a ‘fast Bayesian spectral density approach’ (FBSDA) using the spectral density matrix. In this stage, the ill-conditioning issue of conventional BSDA is tackled. The analysis in this stage reveals that FBSDA can be viewed as the linear superposition of ‘fast Bayesian FFT approach’ incorporating multiple sets of measurements. The companion paper [30] focuses on assembling the local mode shapes corresponding to different setups using the Bayesian statistical framework. The theoretical findings are to be illustrated using simulated data and field data measured from laboratory models equipped with wireless sensors.

2. Revisiting the Bayesian Spectral Density Approach (BSDA)

2.1. Formulation of BSDA

Consider a linear system with n_d dofs subjected to a broadband excitation, such as to ambient vibrations, and assume that discrete acceleration responses are available for $n_o(\le n_d)$ measured dofs and the sampling time interval is assumed to be Δt. Assume that there are n_s sets of independent and identically distributed time histories for these n_o measured dofs. The j-th measured response denoted by $\mathbf{y}_j(n) \in \mathbb{R}^{n_o} (j=1,2,\ldots,n_s)$ at the n-th time step $n\Delta t$ $(n=1,2,\ldots,N)$ is modeled as

$$\mathbf{y}_j(n) = \mathbf{x}_j(n) + \boldsymbol{\mu}_j(n) \tag{1}$$

where $\mathbf{x}_j(n)$ is the j-th model response, a function of the model parameters $\boldsymbol{\lambda}$ to be identified; $\boldsymbol{\mu}_j(n)$ is the j-th prediction error, which can be adequately represented by a discrete zero-mean Gaussian white noise vector process satisfying

$$E[\boldsymbol{\mu}_j(n)\boldsymbol{\mu}_j^T(n')] = \delta_{nn'}\boldsymbol{\Sigma}_{\boldsymbol{\mu}} \tag{2}$$

where $E[\cdot]$ denotes the mathematical expectation; $(\cdot)^T$ denotes the transpose; $\delta_{nn'}$ is the Kronecker delta with $\delta_{nn'}=1$ when $n=n'$, and zero otherwise; $\boldsymbol{\Sigma}_{\boldsymbol{\mu}}$ denotes the covariance matrix of $\boldsymbol{\mu}_j(n)$; the vectors $\mathbf{y}_j(n)$, $\mathbf{x}_j(n)$ and $\boldsymbol{\mu}_j(n)$ are of dimension n_o.

The Fast Fourier transform (FFT) of $\mathbf{y}_j(n)$ at frequency f_k is defined as

$$\mathbf{Y}_j(k) = \mathbf{Y}_{\mathbf{R},j}(k) + \mathbf{i}\mathbf{Y}_{\mathbf{I},j}(k) = \sqrt{\frac{\Delta t}{2\pi N}} \sum_{n=0}^{N-1} \mathbf{y}_j(n)\exp(-\mathbf{i}2\pi f_k n\Delta t) \tag{3}$$

where $\mathbf{i}^2=-1$, $f_k=k\Delta f$, $k=1,2,\ldots,Int(N/2)$, and $\Delta f=1/(N\Delta t)$. In (3), $\mathbf{Y}_{\mathbf{R},j}(k)$ and $\mathbf{Y}_{\mathbf{I},j}(k)$ denote the real and imaginary part of $\mathbf{Y}_j(k)$, respectively. In this work, ‘k’ shown in a bracket or as a subscript denotes the frequency point f_k. The discrete estimator