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Automatic subsystem identification in statistical energy analysis

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ABSTRACT

An automatic methodology for identifying SEA (statistical energy analysis) subsystems within a vibroacoustic system is presented. It consists in dividing the system into cells and grouping them into subsystems via a hierarchical cluster analysis based on the problem eigenmodes. The subsystem distribution corresponds to the optimal grouping of the cells, which is defined in terms of the correlation distance between them. The main advantages of this methodology are its automatic performance and its applicability both to vibratory and vibroacoustic systems. Moreover, the method allows the definition of more than one subsystem in the same geometrical region when required. This is the case of eigenmodes with a very different mechanical response (e.g. out-of-plane or in-plane vibration in shells).

1. Introduction

The solution of acoustic and vibroacoustic interior problems is still nowadays an important challenge for researchers and industry. As discussed in [1], the intrinsic difficulty of the problem is not only the high computational cost of deterministic models, but also the modelling aspects such as the uncertainty of material parameters (specially damping) or the simulation of real boundary conditions. A classical modelling technique in the high-frequency range is the statistical energy analysis (SEA) framework, which is based on the power flow between different parts of the problem domain, called subsystems. A proper subdivision of the domain into weakly coupled subsystems consisting of modes with similar energetic behaviour is crucial for the good performance of SEA. This subdivision combined with modelling aspects such as the correct evaluation of coupling loss factors or the internal losses determines later the quality of the SEA predictions [2]. This paper presents a methodology for identifying the optimal subdivision of a domain into subdomains, such that they can be used as SEA subsystems. The main contributions associated to this methodology are

- The automatic choice of the optimal subdivision (number and geometry of subsystems).
- The applicability of the methodology for problems consisting of both fluid and solid domains.
- The possibility of preprocessing the eigenmodes of the problem depending on their nature. This allows a subsystem definition based on the mechanical behaviour and different subsystems can coexist in the same part of the domain (e.g.

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Nomenclature

e_{ij}	energy density for cell i and mode j
$\tilde{e}_{ij}$	normalised energy density for cell i and mode j
E	Young's modulus
$\overline{E}_j$	mean energy density of the domain for mode j
h	plate thickness
i	cell counter
j	eigenmode counter
n	modal density
N	number of cells
S_j^2	variance of e_{ij} for mode j
$\mathbf{x}_i$	energy vector associated to cell i
Γ_D	Dirichlet boundary
η_{ii}	internal loss factor
η_{ij}	coupling loss factor
λ_{plate}	wavelength at the plate
ν	Poisson's ratio
ρ	density
$\mathbf{1}$	vector with all the components equal to 1
ω	angular frequency

out-of-plane or in-plane vibration in shells).

SEA subsystems are defined by Lyon [3] as “groups of ‘similar’ energy storage modes. These modes are usually modes of the same type (flexural, torsional, etc.) that exist in some section of the system”. In order to classify a certain set of modes as a subsystem, they must fulfil two main criteria [3,2]:

1. *Similarity*: All the modes must have a similar energetic response in front of any possible excitation.
2. *Significance*: They must play an important role in the transmission, dissipation or storage of energy of the problem.

In many typical applications, subsystem identification is straightforward. Common building elements like beams or thin plates clearly fulfil the requirements just discussed. However, SEA may also be a powerful tool for dealing with vibroacoustic problems with complex shapes and non-conventional configurations. Examples of this kind of problem might be found, for instance, in the automotive [4–6], railway [7,8] or aerospace [9] industries, or even in some new architectonic designs and buildings [10]. In these cases the domain subdivision is not so clear and often the decomposition is done following material or geometrical criteria, see Chen et al. [11] or Forssén et al. [7].

Different authors have worked on domain substructuring for energy models. As an example, Kassem et al. [12] propose a strategy for their local vibroacoustic energy model based on searching the validity frequencies for a certain substructuring, and Kovalevsky and Langley [13] propose two different strategies for recognising the elements of their finite element/statistical energy analysis model, based on Green's functions of the problem.

In the particular case of the identification of SEA subsystems, Fahy [14] studies qualitatively the effect of subdividing the cavity inside a car into different subsystems. He concludes that this can be done in particular at the region below the seats, but recommends the use of experimental information for checking the robustness of the approach.

Gagliardini et al. [5] propose a strategy for identifying SEA subsystems. It is based on the energetic transfer functions obtained between points of the domain for different excitations. This analysis involves solving the vibratory problem for every excitation in a particular frequency band.

Totaro and Guyader [15] propose an original strategy based on cluster analysis. It requires the numerical simulation of the vibratory problem for a representative set of excitations. They discretise the domain with finite elements and perform a cluster analysis of these elements. The analysis is based on a set of energy transfer functions obtained for different excitations, and a principal component analysis of these functions is performed before the cluster analysis to reduce the data size. The final decision of the optimal amount of clusters is done in terms of an external parameter called *mutual inertia ratio*.

These methods have only been applied to purely vibratory or purely acoustic systems, but not to vibroacoustic problems. Another limitation of these techniques is that they perform a purely spatial subdivision into subsystems. This means that a certain point of the domain cannot belong to two different subsystems at the same time. However, a certain region of the domain may present two types of modes with very different energetic responses to excitations, as discussed by Lyon [3], Maidanik [16] or McCollum and Cuschieri [17]. For example, a structure consisting of thin shells may present both flexural and in-plane modes for the frequency range of interest.

The strategy presented in this paper is based on a modal analysis of the problem. The domain is divided into small cells, and these cells are classified with a cluster analysis as done in [15]. The difference here is that the analysis is based on the energies for a set of eigenmodes of the problem instead of the frequency-dependent response due to some particular excitation (i.e. point forces). This approach is intimately related to the definition of subsystem proposed by Lyon, and allows as a novelty the possibility of preprocessing the modes for defining more than one subsystem at a certain spatial region. Some added advantages of this strategy are that no excitations are required for the analysis (there are no issues of excitation selection), its applicability to vibroacoustic problems, its low computational cost (it only requires the computation of a few eigenmodes) and the use of the information provided by the cluster analysis to choose the amount of subsystems. Therefore, the choice is independent of any external parameter such as the mutual inertia ratio used in [15]. Moreover, the use of the