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Dynamic modelling of a one-stage spur gear system and vibration-based tooth crack detection analysis

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ABSTRACT

For the purpose of simulation and vibration-based condition monitoring of a geared system, it is important to model the system with an appropriate number of degrees of freedom (DOF). In earlier papers several models were suggested and it is therefore of interest to evaluate their limitations. In the present study a 12 DOF gear dynamic model including a gyroscopic effect was developed and the equations of motions were derived. A one-stage reduction gear was modelled using three different dynamic models (with 6, 8 and 8 reduced to 6 DOF), as well as the developed model (with 12 DOF), which is referred as the fourth model in this paper. The time-varying mesh stiffness was calculated, and dynamic simulation was then performed for different crack sizes. Time domain scalar indicators (the RMS, kurtosis and the crest factor) were applied for fault detection analysis. The results of the first model show a clearly visible difference from those of the other studied models, which were made more realistic by including two more DOF to describe the motor and load. Both the symmetric and the asymmetric disc cases were studied using the fourth model. In the case of disc symmetry, the results of the obtained response are close to those obtained from both the second and third models. Furthermore, the second model showed a slight influence from inter-tooth friction, and therefore the third model is adequate for simulating the pinion's y-displacement in the case of the symmetric disc. In the case of the asymmetric disc, the results deviate from those obtained in the symmetric case. Therefore, for simulating the pinion's y-displacement, the fourth model can be considered for more accurate modelling in the case of the asymmetric disc.

1. Introduction

The vibration-based condition monitoring technique has gained a great deal of importance in the maintenance engineering of industrial gear transmissions. The role of this technique is to detect deterioration, on the basis of the obtained vibration signal, before the occurrence of sudden breakage. Any undetected fault can result in a malfunction and

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then affect the availability of the whole system. Therefore, early fault detection is required to allow proper scheduled maintenance to prevent catastrophic failure and consequently provide safer operation and higher cost savings. Vibration response can be measured experimentally or modelled theoretically. The experimental approach is usually associated with higher costs and problems in accessing the measurement nodes, and is often time-consuming. Furthermore, experimental work is usually restricted in terms of producing enough real faults of desired dimensions. In many cases, therefore, dynamic lumped-parameters modelling can provide us with a clear understanding of the dynamic behaviour of the studied gear system.

Gear modelling can be considered as a fundamental problem which is still the object of much on-going research. A great deal of research has been conducted to study different dynamic models of gear systems [1,2]. Different mathematical gear models were examined in [2], and gear modelling with both torsional and translational vibration was adopted in [3,4]. The one-stage 8 DOF gear dynamic model was applied in [3], while the 6 DOF model was investigated in [4], ignoring the inter-tooth friction. A different 6 DOF gear dynamic model was applied in [5–10]; in this model the friction was considered by simulating 3 DOF for each disc (one torsional and two translational degrees). A one-stage 16 DOF gear dynamic model was developed in [11] and then adopted in [12] for simulating the system dynamic behaviour.

Among the above-mentioned research studies, different dynamic models have been presented for different gear systems. However, there is no study that has examined the influence of adding more DOF to describe the gyroscopic effect of the gear disc. In the present study, a one-stage 12 DOF spur gear model was developed for describing the gyroscopic DOF. This developed model was used to simulate the studied gear system to examine, from a fault detection perspective, if it is necessary to consider the disc asymmetry effect for the studied system. This presented model and three other models were used to simulate the same gear system for different crack sizes. In addition, the present paper explains gear mesh stiffness calculation with a cracked tooth and presents the results of fault detection analysis applied on the dynamic response of the four studied models.

2. Gear dynamic modelling

The modelling of a one-stage reduction gear system is presented in this paper. The main gear parameters were obtained from a real spur gear transmission and are explained in Table 1. This gear transmission is a part of a machine which is used as a test-rig in the Condition Based Maintenance (CBM) Laboratory at Lulea University of Technology. To perform the dynamic simulation, some more parameters need to be introduced in the studied dynamic models, and these parameters are explained in Table 2.

The term ‘pinion’ refers here to the smaller gear, which is a driver gear connected to the input shaft, and the term ‘gear’ refers to the larger gear, which is a driven gear connected to the output shaft. The following notation is used:

m_p/m_g: mass of the pinion/gear;

Table 1
Parameters of the gear–pinion set.

Parameter	Pinion	Gear
Mass (kg)	0.289	1.789
Number of teeth	36	90
Module (mm)	1.5	
Teeth width (mm)	15	
Pressure angle (deg)	20	
Contact ratio	1.76	
Gear ratio	2.5	
Young's modulus, E (N/mm^2)	2×10^5	
Poisson's ratio	0.3	

Table 2
Parameters of the dynamic modelling.

Parameter	Input shaft	Output shaft
Radial stiffness of the bearings in x and y direction (N/m)	6.0×10^8	6.0×10^8
Radial damping of the bearings in x and y direction (N s/m)	1.8×10^3	1.8×10^3
Applied torque (N m)	50	125
Torsional stiffness (N m/rad)	1×10^4	1×10^4
Torsional damping (N m s/rad)	10	10
Rotational speed (Hz)	55.55	22.22
Gear mesh frequency (Hz)	2000	
Coefficient of friction	0.06	