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Inverse investigation of non-Fourier heat conduction in tissue

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ABSTRACT

This paper attempts to describe the heat conduction in tissue using the dual-phase-lag mode. Evaluating the thermo-physical parameters is one of the ways to certify the thermal behavior. As a result, the paper simultaneously and inversely estimates the values of τ_q, τ_T and α for bologna based on the dual-phase-lag mode with the measurement data in the literature. The inconsistence in theory discovered in the literatures is eliminated. The calculated results of temperature variation with the estimated values of τ_q, τ_T and α at the measurement location are very close to the experimental data and address the rationality of the present results.

1. Introduction

Knowledge of temperature distribution in the entire treatment region is essential for controlling the quality of hyperthermia treatment. However, it is difficult to accurately measure tissue temperature and have the temperature field over the entire treatment region during clinical hyperthermia treatments. Therefore, the analysis and modeling of the underlying thermal mechanisms are essential for during development of equipment, for pre-planning purposes, for on-line monitoring and decision support as well as for evaluation of the extent of thermal damage. For simplicity and validity, the Pennes bio-heat transfer equation is commonly used to predict temperature distribution in biological bodies. It bases on the classical Fourier's law that depicts an infinitely fast propagation of thermal signal. Various papers (Luikov, 1968; Braznikov et al., 1975; Kaminski, 1990; Mitra et al., 1995; Roetzel et al., 2003) reported that heat transfer in tissue needs a relaxation time to accumulate enough energy to transfer to the nearest element. The relaxation time in biological tissues is up to 20–30 s (Luikov, 1968; Braznikov et al., 1975). Mitra et al. (1995) did the experimental study of heat transfer in processed meat and reported the thermal relaxation time is of the order of 15 s Roetzel et al. (2003) also made the experimental investigation for processed meat and had the value of relaxation time about 2 s. These studies present the fact that thermal energy transfers in tissue with a limited speed. The Pennes bio-heat model can not seem to completely describe thermal behavior in tissue.

Roetzel et al. (2003), however, further found that the studies (Kaminski, 1990; Mitra et al., 1995) determined the thermal diffusivity (or conductivity) of a material independent of its relaxation behavior while the basic constitutive relation has been changed to a relaxation related one. In other words, the thermophysical quantities were estimated with inconsistence in theory. Based on the measurements for a wide range of materials with non-homogeneous inner structure, Roetzel et al. (2003) ensured the existence of the heat wave behavior in them. But, the values of relaxation time are less than those estimated in Kaminski (1990) and Mitra et al. (1995). Scott et al. (2009) also redid the experiments performed by Mitra et al. (1995) and posted the results. They concluded that the classical Fourier law is enough to describe transient heat conduction behavior in processed meat. However, Scott et al's. (2009 figure 6) shows that the analytical results based on the classical Fourier law do not completely agree with the experimental data at the early times of heating. In fact, non-Fourier thermal behavior is always discovered at the early times of heating (Tzou, 1996; Ho et al., 2003; Xu et al., 2008; Liu, 2008; Liu and Chen, 2009).

For more accurately predicting the variation of temperature in tissue, Antaki (2005) used the dual-phase-lag (DPL) mode to explain the behavior of heat conduction in processed meat for considering the effect of micro-structural interactions. The DPL mode covers a wide scale of space and time for physical observations. Moradi and Ahmadikia (2012) applied it to study the solidification process in living tissue. Askarizadeh and Ahmadikia (2014) used the DPL model to treat the transient heat transfer problems in skin tissue. Kumar et al. (2015) used the finite element wavelet Galerkin method which uses Legendre wavelet as a basis function to the dual-phase-lag model of bio-heat transfer with Gaussian distribution source term under most generalized boundary condition. Narasimhan and Sadasivam (2013) numerically solved the dual-

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Nomenclature

C_1 and C_2	coefficients
c	specific heat of tissue, J/kg °C
$\tilde{F}$	parameter defined in Eq. (13).
k	thermal conductivity, W/m °C
L	height of the containers, m
q	heat flux, W/m^2
s	Laplace transform parameter
t	time, sec
T	temperature of tissue, °C
T_i	initial temperature of tissue, °C
$T_{i,\text{ avg}}$	average initial temperature, °C
$T_{i,r}$	room temperature, °C
$T_{i,c}$	cold temperature, °C
x	distance, m

Greek symbols

α	thermal diffusivity, m^2/s
ε	standard deviation of the measurements
λ	parameter defined in Eq. (12).
ρ	density, kg/m^3
τ_q	phase lag of the heat flux, s
τ_T	phase lag of the temperature gradient, s
$\tilde{\varphi}$	Laplace transform of a function $\phi(t)$

phase-lag Pennes bio-heat equations to predict non-Fourier thermal behavior inside the retinal region of human eye subjected to laser irradiation. In order to explore the DPL behavior of heat transfer in tissue, Liu and Chen (2010) have estimated the relaxation times, τ_q and τ_T, of the muscle tissue from cow based on the experimental data. Yang (2014) proposed a sequential method for estimating the input condition in the DPL model of bio-heat transfer. Jalali et al. (2014) applied the conjugate gradient method as an inverse method to determine the time dependent heat source and the heat transfer coefficient simultaneously in a living tissue.

Evaluating the thermo-physical parameters is one of the ways to certify the thermal behavior. Antaki (2005) predicted the phase lag time of heat flux to be 14–16 s and the phase lag time of temperature gradient to be 0.043–0.056 s for processed meat with the experimental data measured by Mitra et al. (1995). Liu and Lin, (2010) and Liu and Chen (2010) did an extension study for exploring whether the DPL thermal behavior exists in tissue. They estimated the phase lag times in accordance with the experimental data and gave the further evidence to the dual phase lag thermal behavior in tissue. However, more experimental results also are required for showing the physical meanings of the DPL mode in heat transfer in tissue. This paper attempts to describe the behavior of heat conduction in processed meat using the DPL mode. In order to confirm the behavior of the DPL heat conduction, the present paper simultaneously and inversely estimates the values of τ_q, τ_T and α of bologna according to the measurement data given by the literature (Scott et al., 2009). The hybrid application of the Laplace transform and the least-squares scheme is employed to solve the present problem. The temperature variation at the measurement location was calculated with the estimated values of τ_q, τ_T and α and is compared with the experimental data to address the rationality of the present results.

2. Problem description

In order to consider the effect of micro-structural interactions in the transient process of heat transport, Tzou (1995, 1996) introduced a phase lag for temperature gradient, τ_T, absent in the thermal wave model. The corresponding governing model was called the DPL model and was mathematically described as

$$q(x, t + \tau_q) = -k\frac{\partial T(x, t + \tau_T)}{\partial x} \tag{1}$$

where T is the temperature, q the heat flux, k the thermal conductivity, x the distance and t the time. The phase lag of heat flux is τ_q and was used to interpret the short-time thermal inertia which induces the behavior of thermal wave. The phase lag of temperature gradient. is τ_T and was regarded as the effect of micro-structural interactions.

In a local energy balance, the one-dimensional energy equation of the present problem is given as

$$\rho c\frac{\partial T}{\partial t}(x, t) + \frac{\partial q}{\partial x}(x, t) = 0 \tag{2}$$

where ρ and c denote density and specific heat, respectively.

The linearized form of the DPL model is commonly used with the relevant researchers, given as

$$\left(1 + \tau_q\frac{\partial}{\partial t}\right)q = -\left(1 + \tau_T\frac{\partial}{\partial t}\right)k\frac{\partial T}{\partial x} \tag{3}$$

Substituting Eq. (3) into the energy conservation Eq. (2) leads to the DPL equation of heat conduction as the following:

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x} + \tau_T\frac{\partial^2 T}{\partial t\partial x}\right) = \frac{1}{\alpha}\left(1 + \tau_q\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} \tag{4}$$

where α denotes thermal diffusivity.

In order to review the experimental results, Scott et al. (2009) duplicated Experiment I performed by Mitra et al. (1995). The experimental apparatus, please refer to Fig. 2 in (Scott et al., 2009), consists of two identical 13.5 cm high hollow cylindrical plastic containers, closely around the bologna samples. Thermocouples were inserted radially into the samples and at the upper and lower boundaries of each sample. Each of the containers was wrapped with a thin layer of insulation, that is, the sample containers were insulated at the outer circumferences. There was a homogenous initial temperature for each sample, and it was kept at room temperature or a refrigerated temperature, ranging from 3 °C to 13 °C. For the construction of the experimental apparatus, the present problem was regarded as the one-dimensional heat conduction problem.

For theoretical analysis, the conjunction surface between two containers was regarded as the origin surface ($x = 0$), and the boundary temperature was defined as the average initial temperature of the two samples, i.e., $T_{i,avg} = (T_{i,r} + T_{i,c})/2$, where $T_{i,r}$ and $T_{i,c}$ are the room temperature and the cold temperature, respectively (Mitra et al., 1995). As a result, the corresponding boundary conditions are

$$T(0, t) = (T_{i,r} + T_{i,c})/2 \tag{5}$$

$$q(L, t) = 0 \tag{6}$$

and the initial conditions are