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Energy harvesting in a nonlinear piezomagnetoelastic beam subjected to random excitation

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ABSTRACT

This work addresses the influence of nonlinearities in energy harvesting from a piezomagnetoelastic structure subjected to random vibrations. Nonlinear equations of motion that describe the electromechanical system are given along with theoretical simulations. The numerical analysis presents a comparison between the voltage provided from a linear, nonlinear bistable and nonlinear monostable systems due to random vibration. Experimental performance of the generator exhibits qualitative agreement with the theory, showing an enhancement of piezoelectric power generation in a bistable system when it vibrates around both stable equilibrium points. A relationship between variations in the excitation and a bistable system response is established from numerical simulations, defining a region of enhanced power generation when compared to the linear and nonlinear monostable cases.

1. Introduction

Energy harvesting is a concept where available energy is converted into electrical energy that can be used or stored. This idea is very useful for applications in powering small electronic devices. Mechanical vibration energy harvesting can combine this general concept with vibration reduction purposes, eliminating undesirable vibrations to generate useful energy. Piezoelectric elements are essential in this process establishing the mechanical–electrical coupling.

Vibration-based energy harvesting using piezoelectric material has been mainly focused on systems subjected to harmonic excitations. In this case, the best performance is achieved when the system is excited in its fundamental resonance. If the excitation frequency is changed slightly, the power output is drastically reduced. Thus, there have been research efforts focused on the concept of broadband energy harvesting to overcome this drawback [1–5] and some of them explore nonlinear bistable energy converters [6–8]. In this regard, Erturk et al. [9] showed that broadband behavior can be obtained by exploring nonlinearities of a bistable piezomagnetoelastic structure subjected to harmonic excitation. Ferrari et al. [10,11] investigated the same piezomagnetoelastic system studied by Erturk et al. [9] when subjected to random excitation. As in the case with harmonic excited system, the authors showed that the energy harvested could be enhanced by the presence of nonlinearities, depending on the excitation.

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The goal of the analysis presented here is to establish the appropriate excitation to lead to the increase of power output in vibration-based energy harvesting system subjected to random excitations. A piezomagnetoelastic structure under random excitation is investigated considering the influence of nonlinearities. A comparative analysis between linear, nonlinear bistable and nonlinear monostable systems is carried out by both numerical and experimental approaches. Since the bistable system can enhance the harvested energy when the system vibrates around both equilibrium points [10,11], an analysis is carried out with the goal of defining appropriate excitations that lead to an enhanced power output. This desired range is established by defining a relationship between the variance of excitation and the potential energy.

2. Piezomagnetoelastic nonlinear converter

The energy harvesting system is a magnetoelastic structure that consists of a ferromagnetic cantilevered beam with two permanent magnets, one located in the free end of the beam and the other at a vertical distance d from the beam free end, subjected to random base excitation. In order to use this device as a piezoelectric power generator, two piezoceramic layers are attached to the root of the cantilever and a bimorph generator is obtained as depicted in Fig. 1a. The correspondent experimental rig is shown in Fig. 1b.

The PZT layers are connected to an electrical load (a resistor for simplicity) and the voltage output of the generator across the load due to seismic excitation is the primary interest in energy harvesting. The electromechanical system behavior is approximated by the following equations of motion where x is the dimensionless tip displacement of the beam in transverse direction, ζ is the mechanical damping ratio, f_0 is the dimensionless magnitude of excitation and forcing function $n(t)\sim N(1,0)$ is a Gaussian white noise; β and α are respectively, the linear and the nonlinear stiffness coefficients; and an overdot represents differentiation with respect to dimensionless time. Erturk and Inman [12,13] and Erturk et al. [9,14] showed details of the formulation expressed by the following equation:

$$\ddot{x}+2\zeta\dot{x}+\beta x+\alpha x^3-\chi\upsilon=f_0 n(t) \tag{1}$$

$$\dot{\upsilon}+\lambda\upsilon+\kappa\dot{x}=0 \tag{2}$$

where υ is the dimensionless voltage across the load resistance, χ is the dimensionless piezoelectric coupling term in the mechanical equation, κ is the dimensionless piezoelectric coupling term in the electrical circuit equation, and λ is the reciprocal of the dimensionless time constant ($\lambda=1/R_l C_p$, where R_l is the load resistance and C_p is the equivalent capacitance of the piezoceramic layers).

Note that β defines the equivalent beam stiffness and magnet force, while α is defined uniquely by the magnet force. The values of β and α define the representation of different dynamical systems. In terms of equilibrium points, it is possible to observe saddle points, represented by subscript SAD, and stable spiral, represented by subscript SEP. Hence, if $\beta<0$, the system is bistable with three equilibrium points: $(x_{SAD},\dot{x}_{SAD})=(0,0)$ (a saddle) and $(x_{SEP},\dot{x}_{SEP})=(\pm\sqrt{-\beta/\alpha},0)$ (two stable spirals for $0<\zeta<1$). If $\beta\geq 0$ and $\alpha>0$, the system is nonlinear monostable and the equilibrium point is $(x_{SEP},\dot{x}_{SEP})=(0,0)$ (stable spiral). On the other hand, if $\beta\geq 0$ and $\alpha=0$, the system is linear and has the same equilibrium point as the monostable case. These different dynamical systems can be observed by evaluating the potential energy function, presented in Eq. (3), in a range of the tip displacement x, as shown in Fig. 2. Note the difference between single well and double well potentials.

$$U(x)=\frac{1}{2}\beta x^2+\frac{1}{4}\alpha x^4 \tag{3}$$

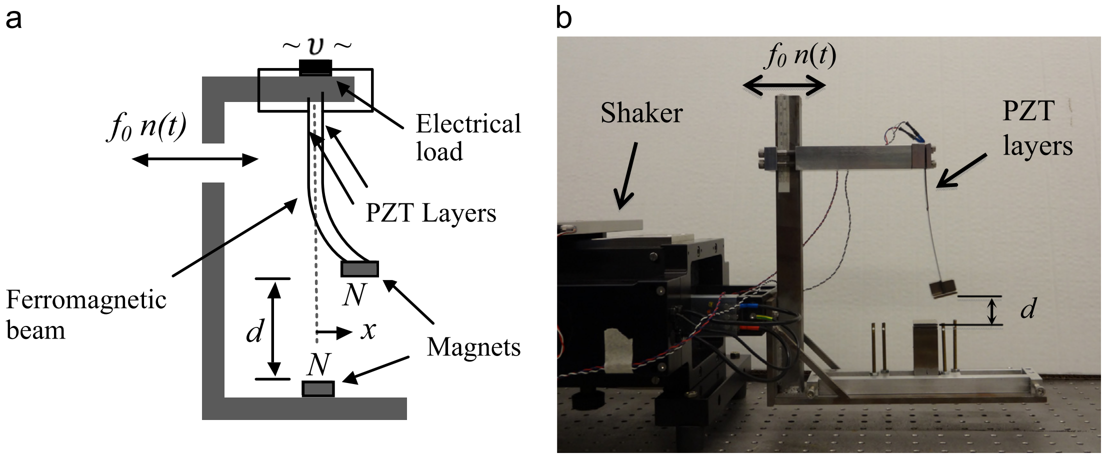

Fig. 1. Piezomagnetoelastic energy harvester set up. (a) Schematic and (b) experimental rig.