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Optimal piezoelectric beam shape for single and broadband vibration energy harvesting: Modeling, simulation and experimental results

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ABSTRACT

Harvesting energy from the surroundings has become a new trend in saving our environment. Among the established ones are solar panels, wind turbines and hydroelectric generators which have successfully grown in meeting the world's energy demand. However, for low powered electronic devices; especially when being placed in a remote area, micro scale energy harvesting is preferable. One of the popular methods is via vibration energy scavenging which converts mechanical energy (from vibration) to electrical energy by the effect of coupling between mechanical variables and electric or magnetic fields. As the voltage generated greatly depends on the geometry and size of the piezoelectric material, there is a need to define an optimum shape and configuration of the piezoelectric energy scavenger. In this research, mathematical derivations for unimorph piezoelectric energy harvester are presented. Simulation is done using MATLAB and COMSOL Multiphysics software to study the effect of varying the length and shape of the beam to the generated voltage. Experimental results comparing triangular and rectangular shaped piezoelectric beam are also presented.

1. Introduction

Energy harvesting has been around for decades. To feed the world's needs for energy, macro scale energy harvesting technologies have successfully established. On the other hand, for low powered electronics devices, harvesting energy from the ambient vibrations seems to be an ideal solution due to the definite life span and high cost for replacement of the traditional batteries. Three mechanisms are available for vibration energy harvesting; using electrostatic devices, electromagnetic field and utilizing piezoelectric based materials [1].

The efficiency of an energy harvester depends on the types of materials used. Piezoelectric energy harvester is widely used in this area of research as it possesses large power densities [2]. Lead Zicronate Titanate (PZT) is preferable due to its abundant vibration accessibility and high piezoelectric constants. Other materials include Lead Magnesium Niobate-Lead Titanate (PMN-PT), Lead Zinc Niobate-Lead Titanate (PZN-PT), Zinc Oxide and Polyvinylidene Difluiride (PVDF).

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Increasing width and thickness of the beam resulted to greater power output [3]. Patel et al. also added that for a piezoelectric material, the influence of its length is greatly dependent on the thickness of the piezoelectric layer [4]. However, in the case of fixed mass, a shorter length, larger width, and lower ratio of piezoelectric layer thickness to total beam thickness are preferred [5].

Truncated beam produces better output voltage as the beam experiences higher stress. Sameh et al. concluded that a trapezoidal cantilever beam will produce more power per unit area compared to a rectangular beam as the distribution of strain is uniform [6]. In addition, cantilever beam is still in favor as it possesses lower resonance frequencies and relatively higher strain for a given force input [7]. However, other non-classical shapes are also being studied in some applications. This includes shell structure [8], spiral [9] and zigzag [10] configurations. Hence, there are a lot of other possibilities that can be explored in order to optimize the power generation of this type of piezoelectric energy harvester.

The main focus of this paper is to study the optimal beam of piezoelectric energy harvester for both single harmonic and broadband vibration. The equations of motions are derived from the Euler-Bernoulli beam theory, and simulation studies are carried out using both MATLAB and COMSOL Multiphysics software. Experimental results are also presented to validate the findings in simulation studies.

2. Free vibration analysis of cantilever beam

In this section, the estimations of natural frequencies for rectangular, trapezoidal and triangular beams are presented. For a uniform beam that undergoes undamped free vibration, the governing equation of motion can be obtained as [11,12]

$$EI\frac{\partial^4 z(x,t)}{\partial x^4}+m\frac{\partial^2 z(x,t)}{\partial t^2}=f_0(x,t) \quad (1)$$

where EI is the bending stiffness, m is the mass per unit length of the beam and $z(x,t)$ is the transverse displacement of the neutral axis (at point x and time t) due to bending. In addition, $z(x,t)$ can be expressed as

$$z(x,t)=z_b(x,t)+z_{rel}(x,t) \quad (2)$$

where $z_b(x,t)$ represents the base motion of the beam and $z_{rel}(x,t)$ denotes the beam's transverse displacement relative to its base.

2.1. Estimation of natural frequencies for rectangular beam

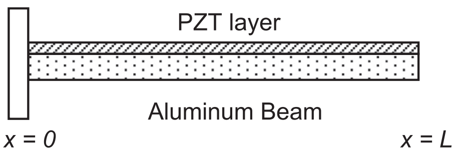

Fig. 1. Piezoelectric energy scavenging system.

The Piezoelectric Energy Scavenging System is pictured in Fig. 1. Using modal analysis technique, the free vibration solution can be expressed as

$$z_{rel}(x,t)=\sum_{k=1}^{\infty} w_k(x)q_k(t) \quad (3)$$

where $w_k(x)$ and $q_k(t)$ are the mass normalized eigenfunction and the modal coordinate of a uniform clamped-free beam of the k-th mode, respectively. Substituting Eq. (3) into Eq. (1), one may obtain

$$\frac{EI}{mw_k(x)}\frac{d^4 w_k(x)}{dx^4}=-\frac{1}{q_k(t)}\frac{d^2 q_k(t)}{dt^2}=\omega_k{}^2 \quad (4)$$

where ω_k^2 is a positive constant so that the response is harmonic. The left term in Eq. (4) is reduced to

$$\frac{d^4 w_k(x)}{dx^4}-\lambda_k^4 w_k(x)=0 \quad (5)$$

where λ_k represents

$$\lambda_k^4=\frac{m}{EI}\omega_k^2 \quad (6)$$

Assuming $w_k(x)=Ce^{sx}$; at which C and s are constants, the solution of Eq. (5) can be expressed as

$$w_k(x)=C_1\sin\left(\frac{\lambda_k}{L}x\right)+C_2\cos\left(\frac{\lambda_k}{L}x\right)+C_3\sinh\left(\frac{\lambda_k}{L}x\right)+C_4\cosh\left(\frac{\lambda_k}{L}x\right) \quad (7)$$