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A revised approach for an exact analytical solution for thermal response in biological tissues significant in therapeutic treatments

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ABSTRACT

The genesis of the present research paper is to develop a revised exact analytical solution of thermal profile of 1-D Pennes' bioheat equation (PBHE) for living tissues influenced in thermal therapeutic treatments. In order to illustrate the temperature distribution in living tissue both Fourier and non-Fourier model of 1-D PBHE has been solved by 'Separation of variables' technique. Till date most of the research works have been carried out with the *constant initial steady temperature of tissue* which is not at all relevant for the biological body due to its nonhomogeneous living cells. There should be a temperature variation in the body before the therapeutic treatment. Therefore, a coupled heat transfer in skin surface before therapeutic heating must be taken account for establishment of exact temperature propagation. This approach has not yet been considered in any research work. In this work, an initial condition for solving governing differential equation of heat conduction in biological tissues has been represented as *a function of spatial coordinate*. In a few research work, initial temperature distribution with PBHE has been coupled in such a way that it eliminates metabolic heat generation. The study has been devoted to establish the comparison of thermal profile between present approach and published theoretical approach for particular initial and boundary conditions inflicted in this investigation. It has been studied that maximum temperature difference of existing approach for Fourier temperature distribution is 19.6% while in case of non-Fourier, it is 52.8%. We have validated our present analysis with experimental results and it has been observed that the temperature response based on the spatial dependent variable initial condition matches more accurately than other approaches.

1. Introduction

The latest amelioration in laser, microwave, ultrasonic technologies have conveyed the phenomena of thermal treatment of diseased organs, injured tissue such as skin burns or skin cancer. The accurate estimation of temperature response within living tissue is very major research interest to provide complete knowledge of design parameters of therapeutic surgical instrument (Field and Bleehen, 1979; Alexander and Griffiths, 1993). For example, the sole objective of hyperthermia is to raise the temperature of diseased tissue up to a standard temperature of 41–56 °C (therapeutic value) by external heat treatment and then thermal destruction takes place without any thermal disturbance of the body (Lagendijk, 2000).

The Pennes' mathematical model (Pennes, 1948) of bioheat transfer is well established due to its simplicity. But this model is based on Classical Fourier's law of heat conduction which states that thermal wave propagates in any domain with an infinite speed. In actual case due to non-uniform and non-homogeneous inner structure of tissues, the propagation of thermal disturbance between blood and tissues always takes place at finite speed. Cattaneo (1958) and Vernotte (1958) concurrently developed C–V model (alternatively single phase lag model) with modification of classical Fourier's hypothesis:

$$q(\vec{r}, t + \tau_q) = -k\nabla T(\vec{r}, t) \quad (1)$$

Eq. (1) demonstrates a time lag between application of heat flux vector and establishment of temperature gradient. The time delay (lag) between heat flux and temperature gradient is defined as thermal relaxation time τ_q which generally (theoretical time scale) ranges from 10^{-4} to 10^{-8} s. The conventional classical Fourier's model of heat conduction covers the macroscopic effects of the domain whereas in practical case after implementation of heat flux, the wave propagation requires certain fraction of time to generate energy and to transform to the nearby element. This is microscopic approach of energy transit in non-homogeneous structure (Tzou, 1996). However, Vedavarz et al. (1994) suggested that phase lag of few biological tissues lies in the range of 1–100 s at room temperature. Kaminski (1990) proposed the

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Nomenclature

A	dimensionless constant defined in Eq. (28).
Bi	Biot no. hL/k, refer Eq. (34a).
c_b	specific heat of blood (J kg^{-1} °C^{-1})
c_p	specific heat of tissue (J kg^{-1} °C^{-1})
C_m, D_m, E_m	dimensionless constant introduced in Eqs. (18), (30) and (39), respectively
F	Fourier number, $\alpha t/L^2$
G	dimensionless constant introduced in Eq. (36).
h	heat transfer coefficient (W m^{-2} °C^{-1})
I, J	dimensionless constant introduced in Eq. (20).
k	effective thermal conductivity of tissue (W m^{-1} °C^{-1})
L	length of the tissue (m)
m	Non negative integers in series (0, 1, 2, 3….)
q_m	metabolic heat generation (J/m^3)
q_s	spatial heat generation (J/m^3)
Q_m^*	dimensionless metabolic heat generation
t	time (s)
T	local temperature of tissue (°C)
T_b	arterial temperature of tissue (°C)
T_i	initial temperature of the tissue (°C)
T_0	skin surface temperature of therapeutic heating (°C)
T_S	steady state temperature of skin tissue (°C)
T_∞	constant initial temperature of skin tissue (°C)
Ve	Vernotte no., $\sqrt{\alpha\tau/L^2}$
x	spatial coordinate staring form skin surface of the tissue (m)
X	dimensionless coordinate, X/L

Greek letters

α	thermal diffusivity of the tissue, $k/\rho c_p$, (m^2 s^{-1})
β	dimensionless blood flow parameter, $\sqrt{\omega_b c_b L^2/k}$
μ_m	Eigen value
ρ	density of the tissue (kg/m^3)
ϕ, ψ	functions used for separation of variables, see Eq. (14).
θ	elevated dimensionless temperature of skin tissue, $T - T_i/T_0 - T_i$
θ_0	dimensionless constant temperature, $T_\infty - T_i/T_0 - T_i$, refer Eq. (12a).
θ_C	dimensionless constant temperature, $T_\infty - T_b/T_0 - T_b$, refer Eq. (24c).
θ_S	dimensionless steady state temperature, defined in Eq. (33).
θ_A	dimensionless constant temperature, $T_0 - T_\infty/T_b - T_\infty$, refer Eq. (37).
θ_{MEAN}	mean dimensionless temperature of tissue, see Eq. (41).
τ	thermal relaxation time (s)
ω_b	blood perfusion rate (kg m^{-3} s^{-1})

relaxation time of 25–30 s for meat products approximately by conducting experiment work.

At very earlier stage, on the basis of PBHE, establishment of analytical models of living tissues by considering the energy interaction and conservation between capillaries and blood vessels (Wulff, 1974; Chen et al., 1981; Weinbaum and Jiji, 1985; Song et al., 1987). In 1990s several authors have applied Green's function method to solve 1-D PBHE problems (Durkee et al., 1990; Durkee and Antich, 1991a, 1991b; Vyas and Rustgi, 1992; Gao et al., 1995; Mitra et al., 1995; Weinbaum et al., 1997; Liu and Xu, 1999; Rai and Rai, 1999; Deng and Liu, 2002).

Shih et al. (2007) investigated thermal response of semi-infinite biological tissues with oscillatory boundary conditions in 1-D form of PBHE omitting metabolic heat generation. Tung et al. (2009) proposed 1-D 'Hyperbolic heat transfer equation' in skin tissue in comparison with conventional Fourier models. Liu (2008) investigated thermal behavior of living tissue subjected to different heating conditions in 1-D manner by implementing Laplace transform technique. Cotta et al. (2010) suggested 'Generalized integral transform technique' for solving 1-D PBHE with linearly variable thermophysical properties of tissue and blood perfusion term in a heterogeneous media. Liu and Chen (2009) presented an elaborated study of mathematical modelling of skin tissue in both Fourier and non-Fourier 1-D models in relation with coupled biological-thermal response under thermal agitation. Gupta et al. (2010) numerically studied heat transfer in biological tissues during thermal therapy by the influence of electromagnetic radiation. Askarizadeh and Ahmadikia (2014) solved 1-D dual phase lag model of bioheat transfer in skin tissue with the help of Laplace transform technique and developed a thorough study to portray the impact of thermal damage. Liu and Chen (2015) illustrated thermal analysis of highly absorbed laser irradiation in living tissues in 1-D dual phase lag form by a hybrid implementation of Laplace transform and discretization method. Kumar et al. (2016a, 2016b) demonstrated temperature distribution on the basis of Laplace transform in different tissues such as muscle, tumor, fat and dermis. A non-linear dual-phase-lag (DPL) bioheat transfer model based on temperature dependent metabolic heat generation rate is established by Kumar et al. (2016a, 2016b) to analyze the heat transfer phenomena in biological tissues during thermal ablation treatment. Lin and Li (2016) portrayed analytical solution of Pennes', C-V model and Dual phase lag model with the help of 'Shifting of variables' method. Kumar et al. (2015) developed numerical solution of 1-D DPL model of bioheat equation by considering initial condition as arterial temperature of the tissue. Recently Kundu (2016) explored 1-D Fourier and non-Fourier model of biological heat transfer for different surface conditions.

From the exclusive literature survey as mentioned above we have attempted to figure out the best possible research work reported in different reputed international journals. We have found a major drawback behind the selection of initial condition for development of analytical solution of heat conduction in living tissue.

As per our vision on the basis of literature review we figured out that:

a. The research papers (Song et al., 1987; Durkee et al., 1990; Durkee and Antich, 1991a, 1991b; Vyas and Rustgi, 1992; Gao et al., 1995; Mitra et al., 1995; Weinbaum et al., 1997; Liu and Xu, 1999; Rai and Rai, 1999; Deng and Liu, 2002; Shih et al., 2007; Tung et al., 2009; Liu, 2008; Cotta et al., 2010; Xu et al., 2009; Gupta et al., 2010; Askarizadeh and Ahmadikia, 2014; Liu and Chen, 2015; Kumar et al., 2016a, 2016b; Lin and Li, 2016; Alkhwaji et al., 2012; Liu et al., 1999; Kumar et al., 2015) deals with the establishment of ***analytical*** mathematical model of biological heat transfer have considered constant initial temperature $T_i(\overrightarrow{S}, 0)$ for the solution. But unlike other engineering problem initial temperature of living tissue can't be assumed as constant. As skin is exposed to environment, temperature gradient always changes due to coupled (conductive-convective) heat transfer. Though the research work (Alkhwaji et al., 2012) depicted the initial condition of skin tissue has not been considered as constant but metabolic heat generation has been omitted. The initial steady state temperature distribution of the skin tissue is obviously independent on time but always dependent on spatial coordinate. The impact of heat transfer coefficient in surrounding fluid near the skin surface should not be negligible while considering steady state temperature distribution. Thus it signifies a serious setback for development of exact temperature